Unsourced Multiple Access (UMAC): Information Theory and Coding

Jean-Francois Chamberland, Krishna Narayanan
Jamison Ebert, Michail Gkagkos
Vamsi Amalladinne, Avinash Vem

International Conference on Signal Processing and Communications (SPCOM)

July 2022

This material is based upon work supported, in part, by NSF under Grants CNS-2148354, CCF-2131106, and CCF-1619085
This material is also based upon work support, in part, by Qualcomm Technologies, Inc., through their University Relations Program
Presentation Outline and Learning Objectives

Unsourced random access
1. Motivation
2. CS and support recovery
3. Inference in large dimensions
4. Sparsifying collisions
5. Data fragmentation
6. Spreading and tensors

Tools for complexity
- Graph-based constructions
- Concatenated codes
- Data fragmentation
- Approximate message passing
- Fast transform methods
- Information propagation

Additional Resources
- PDF slides and source code
- https://engprojects.github.io/mMTC/
- https://github.com/EngProjects/mMTC/ (branch: code)
Fact 1: There are more active cellular subscriptions than there are people

World population: https://www.statista.com/; Cellular subscribers: https://www.worldometers.info/
Visual Acuity and Display Technology

Visual resolution

Peak visual resolution of 20/20 human is

\[
\frac{1}{\text{Visual Acuity}} = \frac{1}{20/20} \text{ min. of arc}
\]

\approx 0.0167 \text{ degrees}

Sharp drops limit viewing angle to \pm 20 degrees

Screen distance

The distance at which super retina HD display matches this resolution

\[
\text{Distance} = \frac{1}{2} \cdot \frac{1}{458} \cdot \cot \frac{1}{120} = 1.876 \text{ in}
\]

**Fact 2:** Display technology is reaching limit of visual acuity
Daily Use of Mobile Devices

Video and mobile statistics

▶ 63% of all US online traffic comes from smartphones and tablets – Stone Temple
▶ More than 70% of YouTube viewing happens on mobile devices – Comscore
▶ 65% of all digital media time is spent on mobile devices – Business2Community

Fact 3 Americans spend significant time on Mobile Devices. Average time spent on mobile phone in US exceeds 3h45m per day – eMarketer
Quality of Experience

Current wireless landscape

▶ **Growth and Market Penetration**: Near saturation
  ▶ Number of connected wireless devices exceeds world population
  ▶ Almost every human who wants mobile phone has one (or more)

▶ **Screen Quality**: At limit of eye acuity
  ▶ Screens are near boundary of visual resolution
  ▶ Viewing distance is constrained by amplitude of accommodation

▶ **Content-Rich Apps**: Video watching & gaming are prevalent
  ▶ On average, a person spends 4 hours on a mobile device per day
  ▶ More videos are watched on phones than elsewhere

What’s Next?
6G Envisioned Traffic Types

- Hyper-Connected Experience: XR, Hologram, Digital Replica
- Ultra-Reliable and Low Latency Communications (URLLC)
- massive Machine Type Communications (mMTC) – Uplink
An Evolving Wireless Landscape

Conventional systems
- Human operators, sustained connections
- Scheduling decisions based on channel quality & queue length
- Acquisition of side information amortized over long connections

Envisioned IoT environments
- Machine-to-machine communications
- Sporadic single transmissions from large number of devices
- Minute payloads
Decentralized Detection in Sensor Networks

Jean-François Chamberland, Student Member, IEEE, and Venugopal V. Veeravalli, Senior Member, IEEE

Abstract—In this paper, we investigate a binary decentralized detection problem in which a network of wireless sensors provides relevant information about the state of nature to a fusion center. Each sensor transmits its data over a multiple access channel. Upon reception of the information, the fusion center attempts to accurately reconstruct the state of nature. We consider the scenario where the sensor network is constrained by the capacity of the wireless channel over which the sensors are transmitting, and we study the structure of an optimal sensor configuration. For the problem of detecting deterministic signals in additive Gaussian noise, we show that having a set of identical binary sensors is asymptotically optimal, as the number of observations per sensor goes to infinity. Thus, the gain offered by having more sensors exceeds the benefits of getting detailed information from each sensor. A thorough analysis of the Gaussian case is presented along with some extensions to other observation distributions.

Index Terms—Bayesian estimation, decentralized detection, sensor network, wireless sensors.

Payload design guideline

▶ Most of information for inference is contained in first few bits!
A Telemetering System by Code Modulation
—Δ-Σ Modulation*

H. INOSE†, MEMBER, IEEE, Y. YASUDA†, AND J. MURAKAMI‡

Summary—A communication system by code modulation is described which incorporates an integration process in the original delta modulation system and is named delta-sigma modulation after its modulation mechanism. It has an advantage over delta modulation in dc level transmission and stability of performance, although both require essentially an equal bandwidth and complexity of circuitry. An experimental telemetering system employing delta-sigma modulation is also described.

Payload design guideline

- Signals are tracked well using small, yet frequent updates
- Δ-Σ modulation

The Principle of the Δ-ΣM System
Losing the Connection

Emerging M2M traffic characteristics

- Device density – Massive versus small
- Connectivity profile – Sporadic versus sustained
- Packet payloads – Minuscule versus moderate-to-long

Anticipated traffic characteristics invalidate the acquisition-estimation-scheduling paradigm!
### Revival of Uncoordinated Access

#### A new reality
- Must address sporadic nature of machine-driven communications
- Transfer of small payloads without ability to amortize cost of acquiring channel and buffer states over long connections
- Preclude use of opportunistic scheduling

#### Communication and identity
When number of devices is massive, with only subset of them active, problem of allocating resources (e.g., codebook, subcarriers, signature sequences) to every user as to manage interference becomes complex

---

**Uncoordinated, Unsourced MAC**
Section Objectives

1. Review connection between unsourced random access and compressed sensing
2. Understand challenges with URA and sparse recovery in exceedingly large dimensional spaces
3. Introduce candidate design strategies to address this research problem
Uncoordinated Multiple Access Channel (MAC)

LoRa-inspired parameters

- $K$ active devices out of $K_{\text{tot}}$, $K \in [25 : 300]$
- Each device has $B$-bit message, $B$ is small $\approx 128$
- $n$ channel uses available, $n \approx 30,000$

Uncoordinated and Unsourced MAC

No personalized feedback
- All devices use same codebook
- No explicit knowledge of identities
- Decoder returns unordered list

Mathematical model
\[ y = \sum_i x_i + z \]
where \( x_i \) depends on message
A perspective on massive random-access

Yury Polyanskiy

Abstract—This paper discusses the contemporary problem of providing multiple-access (MAC) to a massive number of uncoordinated users. First, we define a random-access code for $K_o$-user Gaussian MAC to be a collection of norm-constrained vectors such that the noisy sum of any $K_o$ of them can be decoded with a given (suitably defined) probability of error. An achievability bound for such codes is proposed and compared against popular practical solutions: ALOHA, coded slotted ALOHA, CDMA, and treating interference as noise. It is found out that as the number of users increases existing solutions become vastly energy-inefficient.

MAC [11], [12]). Already 30 years ago R. Gallager [13] called for “a coding technology that is applicable for a large set of transmitters of which a small, but variable, subset simultaneously use the channel.” It appears (to this author) that this call has not been completely answered still. One reason for this could be that the models in each of three categories are different and thus solutions are not directly comparable. Our first goal, thus, is to define a notion of random-access code that would appeal to all three communities. This we do next.

Theorem: Fix $P' < P$. There exists an $(M, n, \epsilon)$ random-access code for the $K$-user GMAC satisfying power-constraint $P$ and

$$\epsilon \leq \sum_{t=1}^{K} \frac{t}{K} \min(p_t, q_t) + p_0,$$

where constants $p_0, p_t, \text{and } q_t$ are complicated.
Uncoordinated MAC Frame Structure

- $K$ active devices out of many, many devices
- Framework of gathering channel and queue states does not apply

Beacon employed for coarse synchronization
- Same set of devices transmit within frame
- Frame may or may not have slots
- Each device may or may not use every slot
URA Framework and Sparse Recovery

Characteristics of URA framework

- Every device employs same codebook $f : \{0, 1\}^B \rightarrow \mathbb{R}^n$
- Decoder must produce unordered list of messages
URA – Compiling Signal Dictionary

Message 00000 maps to column

Message Encoding:

\[ f : \{0, 1\}^B \mapsto \mathbb{R}^n \]

\[ f(\text{binary message}) = \text{signal} \]
URA – Compiling Signal Dictionary

Message 00001 maps to column

Message Encoding:

\[ f : \{0, 1\}^B \mapsto \mathbb{R}^n \]

\[ f(\text{binary message}) = \text{signal} \]
URA – Compiling Signal Dictionary

Message 00010 maps to column $f(00010)$

Message Encoding:

$f : \{0, 1\}^B \mapsto \mathbb{R}^n$

$f(\text{binary message}) = \text{signal}$
URA – Compiling Signal Dictionary

Message 00011 maps to column

Message Encoding:

\[ f : \{0, 1\}^B \mapsto \mathbb{R}^n \]

\[ f(\text{binary message}) = \text{signal} \]
URA – Compiling Signal Dictionary

Message 11111 maps to column

Message Encoding:

\[ f : \{0, 1\}^B \mapsto \mathbb{R}^n \]

\[ f(\text{binary message}) = \text{signal} \]
URA – Index Representation

CS-Style Format:

\[ \text{signal} = \Phi m \]
URA – Index Representation

Binary message 00000011

Message index 00010...0

Codeword (modulated signal)
Unsourced Random Access – CS Analogy

Message 1 ➔ Encoder
Message 2 ➔ Encoder
Message 3 ➔ Encoder
... ➔ ...
Message K ➔ Encoder

Multiple Access Channel ➔ Joint Decoder
Abstract CS Challenge

Problem setting

- Noisy compressed sensing
  \[ y = \Phi s + z \]
  where \( s \) is \( K \) sparse
  - \( s \) has non-negative integer entries
  - \( \Phi.\text{shape} \approx 32,768 \times 2^{128} \)
  - \( z \) is additive Gaussian noise

Practical issues

- Width of sensing matrix is huge
- Existing CS solvers will not execute at that scale
Compressed Sensing – Undersampling

Number of columns

Number of samples

Number of non-zero entries

Sensing matrix $\Phi$

Sparse vector $s$

Observation $y$

Undersampling fraction $\delta$: 

$$\frac{\text{height of } \Phi}{\text{width of } \Phi} = \frac{n}{N} \rightarrow \delta$$
Compressed Sensing – Sparsity

Measure of sparsity $\rho$:

\[
\frac{\text{Signal complexity } \|s\|_0}{\text{height of } \Phi} = \frac{K}{n} \rightarrow \rho
\]
Compressed Sensing – Phase Transitions

Undersampling fraction

$$\delta = \frac{n}{N} = \frac{32,768}{2^{128}} = 2^{-113}$$

Measure of sparsity

$$\rho = \frac{K}{n} = \frac{256}{32,768} = 2^{-7}$$
Classical Coding Techniques

Multi-user coding

- Matrix becomes codebooks
  \[ y = \Phi_1 s_1 + \Phi_2 s_2 + z \]
- Device picks code based on bits
- Well-studied for single user
- Fast decoding for large dictionary

Drawbacks

- Low complexity joint multi-user decoders are not available
- Devices may collide within codebook selection
**Time-Division Unsourced Random Access**

**Slot partitioning**
- Observations become
  \[ y_\ell = \Phi_\ell s_\ell + z_\ell \]
  where \( \ell \) is slot label
- Device gets slot based on message
- Channel uses divided among slots

**Drawbacks**
- Matrices remain wide \( 2^{128} / \# \) slots
- Devices assigned randomly within slots
Quest for Low-Complexity Unsourced MAC

Sparsifying collision via stochastic binning

$n$ symbols

Slot 0  Slot 1  Slot 2  Slot 3  Slot 4  Slot 5

Random Selection

Codewords

Messages

Caveat – The Poisson Wall

Effects of decoding threshold

- More slots reduces parameter of Poisson/binomial distribution
- More slots reduces bit count per decoded slot

\[ \sum_{k=0}^{T} \frac{N}{J} \frac{k}{T} \log_2 (1 + JT \cdot \text{SNR}) \text{ pmf}(k) \]
Data Fragmentation

Drawbacks

▶ Unordered lists of fragments
▶ Need to perform disambiguation
Section Summary

Problem formulation

- Noisy compressed sensing
  \[ y = \Phi s + z \]
- URA is noisy support recovery
- Full control over \( \Phi \)
- Width of sensing matrix is huge
- Uncoordinated access produces stochastic binning

Possible URA design strategies

- Sparsifying collisions
- Advanced coding and spreading
- Data fragmentation
Pertinent References


Part I

Sparsifying Collisions: Graph-Based Techniques and Concatenated Coding
Sparsifying Collisions

Graphs

- Quest for low complexity URA (mMTC) schemes
- Draw inspiration from graphical models
- Belief propagation
- Survey past successes 3 ways
  1. LDPC/LDGM codes
  2. Compressed sensing
  3. Random access

Section goal

Gain ability to create graph-based URA schemes
Sparse Graph: Tools from Iterative Decoding

- $L_i$ variable dist. from node
- $\lambda_i$ variable dist. from edge

- $R_j$ check dist. from node
- $\rho_j$ check dist. from edge

Sparse Graph: Computation Tree

Standard tricks

- Unravel bipartite graph
- Graph needs to be locally tree-like
- Focus on outgoing messages
- Analyze over random code ensemble

Sparse Graph: Analyzing Iterative Decoding

- $x$: Prob. outgoing message from variable node erased
- $y$: Prob. outgoing message from check node erased

- Outgoing variable message erased if all incoming check messages are erased
  \[ x = E[y^{i-1}] = \lambda(y) \]

- Outgoing check message erased if any incoming variable message is erased
  \[ y = E[1 - (1 - x)^{j-1}] = 1 - \rho(1 - x) \]

---
Extrinsic Information Transfer (EXIT) chart

\[ y = 1 - \rho(1 - x) \quad \text{and} \quad x = \lambda(y) \quad \text{(flipped)} \]
Sparse Graph Code Based Compressed Sensing

Support recovery problem

\[ y = \Phi x + z \]

- Sensing matrix \( \Phi \) is \( n \times N \)
- Variable \( z \) is additive noise
- Recover \( \text{supp}(x) = \{i : x_i \neq 0\} \)

Objective

Devise scheme with minimal number of measurements \( n \) and minimal decoding complexity such that \( \Pr(\text{failure}) \to 0 \) as \( N \) (and \( K \)) \to \infty \)
Support Recovery – Fundamental Limit

Optimal order for support recovery

- In the sub-linear sparsity regime, $K = o(N)$, necessary and sufficient conditions are shown to be:

$$C_1 K \log \left( \frac{N}{K} \right) < n < C_2 K \log \left( \frac{N}{K} \right)$$

- In the linear sparsity regime, $K = \alpha N$, it was shown that $n = \Theta(N)$ measurements are sufficient for asymptotically reliable recovery.

- Minimum value $x_{\text{min}}$ has to be bounded away from zero

---


Sparse Graph Code Based Compressed Sensing

- Sparse-graph codes peeling decoder framework for CS
- Sample and measurement complexities of order $O(K \log N)$ for noisy setting\(^1\)
- Complexities of $O(K \log N/K)$ for noisy setting\(^2\)

Biadjacency matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$


Tensoring Construction

\[ A = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix} \]

\[ S = \begin{bmatrix}
+1 & -1 & -1 \\
-1 & +1 & -1
\end{bmatrix} \]

Sensing matrix is tensor-inspired product

\[
A \boxtimes S = \begin{bmatrix}
+1 & 0 & 0 & -1 & 0 & -1 \\
-1 & 0 & 0 & +1 & 0 & -1 \\
0 & +1 & -1 & 0 & -1 & 0 \\
0 & -1 & +1 & 0 & -1 & 0 \\
+1 & -1 & 0 & -1 & 0 & 0 \\
-1 & +1 & 0 & -1 & 0 & 0 \\
0 & 0 & +1 & 0 & -1 & -1 \\
0 & 0 & -1 & 0 & +1 & -1
\end{bmatrix}
\]

Design amenable to peeling decoding when \( x \) is real vector
Random Access with Twist

System model

- **K uncoordinated** devices, each with 1 packet to send
- **Time is slotted**; transmissions occur within slots
- Receiver knows full schedule, collection of packets in every slot
- **Successive interference cancellation**

---

Graphical Representation

- Tanner graph representation for LDGM transmission scheme
- Info nodes ↔ packets; Coded nodes ↔ received signals
- Message-passing (SIC) – **peeling decoder** for erasure channel

---


Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Instance of Random Access
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 1
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 1
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 2
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 2
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 3
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 3
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

Step 4
Representations: Schedule, Tanner Graph, Compressed
Sparse Graph – Back to Unsourced Random Access

- Sensing Matrix $n \times N$
- $N = 2^{128}$ columns
- $K \approx 100$ active devices
- Non-negative coefficients
- $n \approx 30,000$ measurements
- Complexity $\mathcal{O}(K \log N)$
- Support recovery

\[
\begin{array}{c}
\text{Received Signal} \\
\vdots \\
\text{Message indices}
\end{array}
\]
URA – A Quest for Low Complexity

Partition into $J$ slots

$\tilde{n} = n/J$ channel uses

Aim is $T$-user adder channel

Admits graphical representation
Schedule selected based on **message**

- Devices can transmit in multiple sub-blocks
- Scheme facilitates successive interference cancelation

---

What Really Happens within Slot?

Message is partitioned into two parts $w = (w_1, w_2)$

Every device uses identical codebook built from LDPC-type codes tailored to $T$-user real-adder channel

$w_1$ is encoded with a spatially-coupled LDPC code and then permuted based on $w_2$

$w_2$ is compressed via CS matrix $A$ and recovered through non-negative $\ell_1$-regularized LASSO
Sparse Unsourced Random Access

- Compressed sensing preamble with information bits
- Sparse graph-based random access scheme conducive to joint decoding
Sparsifying Collision with Graph-Based Techniques

- Minimum $E_b/N_0$ required as function of number of devices
- For $T = 2, 4$ and 4-fold ALOHA, prob. of decoding every slot $\geq 0.99$
- Prob. recovered messages $\geq 0.96$ given $T$-user decoding successful
Background and Pertinent References

Part II

Data Fragmentation:
A Divide-and-Conquer Approach
to Compressed Sensing
CCS – Fragmentation with Disambiguation

Stitching through outer code

- Split problem into sub-components suitable for CS framework
- Get lists of sub-packets, one list for every slot
- Stitch pieces of one packet together using error correction
CCS – Sensing Matrix, Single-User Indexing

Number of columns $2^B$

Number of samples

Width of matrix is HUGE!
SPARC reduces width drastically.

8-SPARC: \(2^{128}\) becomes \(8 \cdot 2^{16}\)
Drastic Reduction in Matrix Width

- Undersampling fraction
  \[ \delta = \frac{n}{N} = \frac{32,768}{L \cdot 2^{128/L}} \]

- Measure of sparsity
  \[ \rho = \frac{K}{n} = \frac{L \cdot 256}{32,768} = \frac{L}{2^7} \]
Coded Compressive Sensing – Device Perspective

Information bits

Outer code

Coupled messages

Slot 1

Slot 2

Slot 3

Slot $L$

- Collection of $L$ CS matrices and 1-sparse vectors
- Each CS generated signal is sent in specific time slot

---

L instances of CS problem, each solved with non-negative LS

- Produces $L$ lists of $K$ decoded sub-packets (with parity)
- Must piece sub-packets together using tree decoder
Coded Compressive Sensing – Stitching Process

Tree decoding principles

- Every parity is linear combination of bits in preceding blocks
- Late parity bits offer better performance
- Early parity bits decrease decoding complexity
- Correct fragment is on list
Coded Compressive Sensing – Divide and Conquer

- Data fragmentation and indexing
- Outer encoding for disambiguation
Vignette – Compressed Sensing

Sampling complexity:

\[ c \cdot K \log \frac{N}{K} \]
Optimization objective with sparsity constraint

When $\Phi$ satisfies certain conditions, e.g., RIP, we can get a good estimate for sparse $s_0$ by solving convex program

$$\hat{s} = \arg \min_s \|y - \Phi s\|_2 + \lambda \|s\|_1$$

- Extensive analysis and wide applications
- LP, QP, ISTA w/o momentum, NNLS, etc.
Optimization task

\[
\begin{align*}
\text{minimize} & \quad \|s\|_2 \\
\text{subject to} & \quad \Phi s = y
\end{align*}
\]

Composite iterative solution

\[
\begin{align*}
z^{(t)} &= y - \Phi s^{(t)} \\
s^{(t+1)} &= \Phi^T z^{(t)} + s^{(t)}
\end{align*}
\]

Analysis

\[
\mathcal{R}(\Phi) \xrightarrow{\Phi^T} \mathcal{N}(\Phi^T)
\]

Synthesis

\[
\mathcal{N}(\Phi^T) \xrightarrow{\Phi} \mathcal{R}(\Phi)
\]

Neumann series

\[
s^{(t+1)} \to \Phi^T (\Phi \Phi^T)^{-1} y
\]
Approximate message passing (AMP)

\[ z^{(t)} = y - \Phi s^{(t)} + \frac{z^{(t-1)}}{n} \|s^{(t)}\|_0 \]

\[ s^{(t+1)} = \eta(\Phi^T z^{(t)} + s^{(t)}) \]

where \( \eta(s)_k = (|s_k| - \alpha \lambda)_+ \text{sgn}(s_k) \), \( s^{(0)} = 0 \), \( z^{(0)} = y \)

- Application to high-dimensional spaces
- Low complexity, scalable framework
Approximate message passing (AMP)

\[
\begin{align*}
\mathbf{z}^{(t)} &= \mathbf{y} - \mathbf{\Phi}s^{(t)} + \frac{\mathbf{z}^{(t-1)}}{n} ||s^{(t)}||_0 \\
\mathbf{s}^{(t+1)} &= \eta(\mathbf{\Phi}^T\mathbf{z}^{(t)} + \mathbf{s}^{(t)})
\end{align*}
\]

where \(\eta(s)_k = (|s_k| - \alpha \lambda)_+ \text{sgn}(s_k), \ s^{(0)} = 0, \ z^{(0)} = \mathbf{y}\)

- Application to high-dimensional spaces
- Low complexity, scalable framework
CCS – Approximate Message Passing

SPARC for Unsourced Random Access

Alexander Fengler, Peter Jung, Giuseppe Caire

(Submitted on 18 Jan 2019)

This paper studies the optimal achievable performance of compressed sensing based unsourced random–access communication over the real AWGN channel. "Unsourced" means, that every user employs the same codebook. This paradigm, recently introduced by Polyanskiy, is a natural consequence of a very large number of potential users of which only a finite number is active in each time slot. The idea behind compressed sensing based schemes is that each user encodes his message into a sparse binary vector and compresses it into a real or complex valued vector using a random linear mapping. When each user employs the same matrix this creates an effective binary inner multiple–access channel. To reduce the complexity to an acceptable level the messages have to be split into blocks. An outer code is used to assign the symbols to individual messages. This division into sparse blocks is analogous to the construction of sparse regression codes (SPARCs), a novel type of channel codes, and we can use concepts from SPARCs to design efficient random–access codes. We analyze the asymptotically optimal performance of the inner code using the recently rigorized replica symmetric formula for the free energy which is achievable with the approximate message passing (AMP) decoder with spatial coupling. An upper bound on the achievable rates of the outer code is derived by classical Shannon theory. Together this establishes a framework to analyse the trade–off between SNR, complexity and achievable rates in the asymptotic infinite blocklength limit. Finite blocklength simulations show that the combination of AMP decoding, with suitable approximations, together with an outer code recently proposed by Amalladinne et. al. outperforms state of the art methods in terms of required energy–per–bit at lower decoding complexity.

Comments: 16 pages, 7 Figures
Subjects: Information Theory (cs.IT)
Cite as: arXiv:1901.06234 [cs.IT]
(or arXiv:1901.06234v1 [cs.IT] for this version)

▶ Connection between CCS indexing and sparse regression codes
▶ Circumvent slotting under CCS and dispersion effects
▶ Introduce denoiser tailored to CCS
CCS Revisited

- Bit sequence split into $L$ fragments
- Each bit + parity block converted to index in $[0, 2^{B/L} - 1]$
- Stack sub-codewords into $(n/L) \times 2^{B/L}$ sensing matrices

Columns are possible signals
Coded Compressed Sensing – Unified View

- Slots produce block diagonal (unified) matrix
- Message is one-sparse per section
- Width of $A$ is smaller: $L^{2B/L}$ instead of $2^B$
CCS – Full Sensing Matrix

- Complexity reduction due to narrower $A$
- Use full sensing matrix $A$
- Decode inner code with low-complexity AMP

Received Signal

\( y \)

Input

\( z(t) \)

\( A \)

Delay

Effective Observation

\( A^T \)

\( r(t) \)

\( s(t) \)

Delay

Denoiser

\( \mathbb{E}[s | s + \tau \zeta = r] \)

PME

\( \frac{1}{n} \text{div}(\cdot) \)

Onsager Term

Output
## Governing equations

- **AMP algorithm iterates through**

\[
\begin{align*}
    z^{(t)} &= y - A D \eta_t (r^{(t)}) + \frac{z^{(t-1)}}{n} \text{div} D \eta_t (r^{(t)}) \\
    r^{(t+1)} &= A^T z^{(t)} + D \eta_t (r^{(t)})
\end{align*}
\]

- **Onsager correction**

- **Denoiser**

- **Initial conditions** \( z^{(0)} = 0 \) and \( \eta_0 (r^{(0)}) = 0 \)

- **Application falls within framework for non-separable functions**

## Task

- **Define denoiser and compute Onsager correction term**
Marginal Posterior Mean Estimate (PME)

Proposed denoiser (Fengler, Jung, and Caire)

- State estimate based on Gaussian model

\[
\hat{s}^{\text{OR}}(q, r, \tau) = \mathbb{E} \left[ s \left| \sqrt{P_\ell} s + \tau \zeta = r \right. \right]
\]

\[
= \frac{q \exp \left( - \frac{(r - \sqrt{P_\ell})^2}{2\tau^2} \right)}{(1 - q) \exp \left( - \frac{r^2}{2\tau^2} \right) + q \exp \left( - \frac{(r - \sqrt{P_\ell})^2}{2\tau^2} \right)}
\]

with (essentially) uninformative prior \( q = 1 - (1 - \frac{1}{m})^K \) fixed

- \( \eta_t(r^{(t)}) \) is aggregate of PME values

- \( \tau_t \) is obtained from state evolution or \( \tau_t^2 = \|z^{(t)}\|^2 / n \)
Performance of CCS Schemes

Incorporating Lessons from Enhanced CCS

- Integrate outer code structure into inner decoding

**Challenges**

- CCS-AMP inner decoding is not a sequence of hard decisions
- List size for CCS-AMP is effective length of index vector

CCS – AMP Architecture with Outer Code
Redesigning Outer Code

Properties of original outer code

- Aimed at stitching message fragments together
- Works on short lists of $K$ fragments
- Parities allocated to control growth and complexity

Challenges to integrate into AMP

1. Must compute beliefs for all possible fragments
2. Must provide pertinent information to inner AMP decoder
3. Should maintain ability to stitch outer code
Factor Graph Interpretation of Outer Code

Outer code with circular convolution structure

\[
\mu_{ap \rightarrow s_\ell} ([\hat{v}(\ell)]_2) \propto \frac{1}{\|g_{\ell,p}\|_0} \left( \text{FFT}^{-1} \left( \prod_{s_j \in N(ap) \setminus s_\ell} \text{FFT} (\lambda_j, p) \right) \right) (g)
\]
Multiple devices on same graph
- Parity factor mix concentrated values
- Suggests triadic outer structure
Redesigning Outer Code

Solutions to integrate into AMP

- Parity bits are generated over Abelian group amenable to FWHT or FFT
- Discrimination power proportional to \# parities

New design strategy

1. Information sections with parity bits interspersed in-between
2. Parity over two blocks (triadic dependencies)
Belief Propagation – Message Passing Rules

▶ Message from check node $a_p$ to variable node $s \in N(a_p)$:

$$\mu_{a_p \to s}(k) = \sum_{k:p=k} G_{a_p}(k_{a_p}) \prod_{s_j \in N(a_p) \setminus s} \mu_{s_j \to a_p}(k_j)$$

▶ Message from variable node $s_\ell$ to check node $a \in N(s)$:

$$\mu_{s_\ell \to a}(k) \propto \lambda_\ell(k) \prod_{a_p \in N(s_\ell) \setminus a} \mu_{a_p \to s_\ell}(k)$$

▶ Estimated marginal distribution

$$p_{s_\ell}(k) \propto \lambda_\ell(k) \prod_{a \in N(s_\ell)} \mu_{a \to s_\ell}(k)$$
Approximate Message Passing Algorithm

Updated equations

AMP two-step algorithm

\[ z^{(t)} = y - AD\eta_t(r^{(t)}) + \frac{z^{(t-1)}}{n} \text{div } D\eta_t(r^{(t)}) \]

Correction

\[ r^{(t+1)} = A^Tz^{(t)} + D\eta_t(r^{(t)}) \]

Denoiser

Initial conditions \( z^{(0)} = 0 \) and \( \eta_0(r^{(0)}) = 0 \)

- Denoiser is BP estimate from factor graph
- Message passing uses fresh effective observation \( r \)
- Fewer rounds than shortest cycle on factor graph
- Close to PME, but incorporating beliefs from outer code

Performance improves significantly with enhanced CCS-AMP decoding

Computational complexity is approximately maintained

Reparametrization may offer additional gains in performance?
CCS and AMP Summary

Summary

- New connection between CCS and AMP
- Natural application of BP on factor graph as denoiser
- Outer code design depends on sparsity
  1. Degree distributions (small graph)
  2. Message size (birthday problem)
  3. Final step is disambiguation
- Many theoretical and practical challenges/opportunities exist

Coding plays increasingly central role in large-scale CS
CCS via Coded Demixing

- CCS can be extended to accommodate multiple classes of heterogeneous users
- Each class of users employs its own sensing matrix and factor graph for message encoding
- Every class transmits its signal concurrently over GMAC
Through coded demixing, signals from various classes may be separated by receiver and decoded individually.

Efficient AMP-based algorithm to recover signals from different classes.

Requires signals to be sparse and the sensing matrices to have low cross-coherence.
Coded Demixing for Single-Class URA

- Create multiple bins with (incoherent) matrices
- Devices pick a bucket randomly and use CCS-AMP encoding
- Perform joint demixing CCS-AMP decoding at access point

PUPE = 5%

CCS – Coded Demixing Architecture

Received Signal

Input

y

\[ \sum \]

z^{(t)}

A

\[ \times \]

\[ \frac{1}{n} \text{div}(\cdot) \]

D

Amplitude

Delay

Stack

Output

PME

Delay

Dynamic PMEs

Delay

\[ s^{(t)}_1 \]

\[ s^{(t)}_2 \]

\[ s^{(t)}_3 \]

\[ r^{(t)}_1 \]

\[ r^{(t)}_2 \]

\[ r^{(t)}_3 \]

Delay
Performance of CCS-AMP versus Previous Schemes

- Sparse interleave division multiple access (IDMA) by A. K. Pradhan, V. Amalladinne, A. Vem, K. R. Narayanan and JFC
- Sparse Kronecker-product (SKP) coding by Z. Han, X. Yuan, C. Xu, S. Jiang and X. Wang


Pertinent References

Part III

Sparsifying Collisions:
Spread Unsourced Random Access
with Tensor/Hadamard Constructions
Spreading or Stochastic Binning with Shadowing

- Message is partitioned into two parts \((p, m)\)
- Preamble \(p\) selects spreading sequence
- Payload \(m\) is encoded using traditional code: Polar/LDPC
- Tensor/Hadamard product is performed to create signal

Note: \(c_k \otimes s_{\hat{k}}\) or \((c_k \otimes 1) \circ s_{\hat{k}}\)
Signal Structure and Energy Detector

- Outer product representation of sent signal

\[ X = s \cdot c^T = \begin{bmatrix} \begin{array}{c} s \cdot c_1 \\ s \cdot c_2 \\ \vdots \\ s \cdot c_m \end{array} \end{bmatrix} \quad s^H X = \|s\|_2^2 c^T \]

- Outer product representation of received signal

\[ Y = \sum_k X_k + Z = \sum_k s_k^* \cdot c_k^T + Z \]

\[ s_k^H Y = \|s_k\|_2^2 c_k^T + \sum_{\ell \neq k} \langle s_k^*, s_{\ell} \rangle c_{\ell}^T + s_k^H Z \]

- Energy detector for sequence identification

Set of active sequence = $\text{top arg} \|s_k^H Y\|_2$
Joint Decoding Architecture

1. Sequence identification
2. Symbol estimation
3. Bank of single-user decoders
4. Signal reconstruction
5. Successive interference cancellation
Intuition Behind Symbol Estimation

Measurement structure (w/o collisions)

\[
\begin{align*}
Y &= \begin{bmatrix}
y_1 & y_2 & \cdots & y_m
\end{bmatrix} \\
&= \sum_k \begin{bmatrix} s_k \end{bmatrix} \begin{bmatrix} -c_k \end{bmatrix} + Z \\
&= SC + Z
\end{align*}
\]

Approximate structure

\[
Y \approx S_D C + Z
\]

\[
\approx \sum_{k \in D} \begin{bmatrix} s_k \end{bmatrix} \begin{bmatrix} -c_k \end{bmatrix} + Z
\]

covariance

\[
(S_D S_D^H + I)
\]

LMMSE

\[
\hat{C} \approx S_D^H (S_D S_D^H + I)^{-1} Y
\]
Spreading or Stochastic Binning with Shadowing

- Polar code
- Single-user likelihoods based on estimated rows of
  \[ \hat{C} \approx S_D^H (S_D S_D^H + I)^{-1} Y \]
- Joint successive cancellation within decoding loop
- CRC added to codewords
- Sequences dictate frozen bits
- LMMSE can be tuned to account for collisions
- Framework can accommodate soft LDPC symbol estimates
Spread polar outperforms Irregular Repetition Slotted ALOHA (IRSA) polar by E. Marshakov, G. Balitskiy, K. Andreev, A. Frolov

Low complexity scheme by D. Truhachev, M. Bashir, A. Karami, E. Nassaji performs well for large population
Signal Structure Revisited

- Note: \( c \otimes s \) or \( (c \otimes 1) \circ s^+ \)
- New representation of sent signal

\[
X = (c \otimes 1) \circ s^+ \Rightarrow \begin{bmatrix} s^+(1) c_1 & s^+(2) c_2 & \cdots & s^+(m) c_m \end{bmatrix}
\]

with a different spreading column for every coded symbol

- One LMMSE matrix inversion per coded symbol period \( j \)

\[
\hat{c}(j) \approx S_{j,D}^H (S_{j,D}S_{j,D}^H + I)^{-1} y_j
\]

- Framework becomes more general and subsumes IRSA
- Match filter versus LMMSE
  - Buying performance at expense of complexity
  - Model becomes more brittle to fine synchronization
- Random subset of sequence set precludes CDMA-style designs
Part IV

Quasi-Static Massive MIMO
Unsourced Random Access Channels
Massive MIMO URA – Quasi-Static Channel

Quasi-static signal model

- Signal received at time instant $t$ within slot $\ell$

$$y(t, \ell) = \sum_{k=1}^{K} x_k(t, \ell) h_k + z(t, \ell)$$

- Number of receive antennas $M \gg 1$
- Fading does NOT change within URA frame
Massive MIMO URA – Quasi-Static Channel

Problem formulation

- Noisy MMV support recovery

\[ \mathbf{Y} = \sum_k \mathbf{x}_k \cdot \mathbf{h}_k^T + \mathbf{Z} \]

- Channel coefficients are not known
- Number of antennas \( M \) is large
- Channel is quasi-static

Possible URA design strategies

- Collisions may not be as much of an issue
- Complexity must be managed
- Strategies with pilots seem advantageous
Proposed Encoding – Pilot plus Spreading

- Encoding similar to spread URA, albeit with pilots
- Pilot sequence used for activity detection and channel estimation
- Payload $m$ is encoded using traditional code: Polar/LDPC
Joint Decoding Architecture MIMO

\[
\hat{Y} = \sum_{k \in D} \hat{x}_k \cdot \hat{h}_k^T
\]

Energy Detector

Pilot set = \text{top arg} \| p_k^H Y_p \|_2

LMMSE channel estimation

\[
\hat{H} = \left( \sigma^2 I + \hat{P}^H \hat{P} \right)^{-1} \hat{P}^H Y_p
\]
Intuition Behind MIMO Symbol Estimation

Measurement structure
For symbol period $j$

\[
Y[n_j, m] \approx S_j \text{diag}(c_j)\hat{H}[:, m] + Z[n_j, m]
\]

\[
= S_j \text{diag} \left( \hat{H}[:, m] \right) c_j + Z[n_j, m]
\]

where $n_j = [(j - 1)L + 1: jL]$ spread slice and $m$ is antenna index

Stacked vector and LMMSE estimates

\[
\begin{bmatrix}
Y[n_j, 1] \\
\vdots \\
Y[n_j, M]
\end{bmatrix}
= \begin{bmatrix}
S_j \text{diag} \left( \hat{H}[:, 1] \right) \\
\vdots \\
S_j \text{diag} \left( \hat{H}[:, M] \right)
\end{bmatrix} c_j + \begin{bmatrix}
Z[n_j, 1] \\
\vdots \\
Z[n_j, M]
\end{bmatrix}
\]
## Alternate Scheme – Tensor-Based Modulation

### Code Construction

Codebook is created based on tensors

$$\left\{ x_1 \otimes x_2 \otimes \cdots \otimes x_d : x_1 \in C_1, x_2 \in C_2, \ldots, x_d \in C_d \right\}$$

Received signal is sum of $K$ rank-1 tensors plus noise

$$\sum_{k} x_{1,k} \otimes x_{2,k} \otimes \cdots \otimes x_{d,k} \otimes h_k + z$$

- Decode with canonical polyadic decomposition (CPD)
- Iterative nonlinear least square algorithm on flattened outer products
- Pilots are not used in this scheme

---

Alternate Scheme – Orthogonal Pilots

- Hadamard pilots for fast processing: detection and estimation
- Polar code plus cyclic redundancy check (CRC)
- Excellent performance versus complexity tradeoff

Spread URA Single-Antenna

- TBM w/o pilots, $P_e = 0.1$
- Pilot/MF, $P_e = 0.05$
- Orthogonal Pilots, $P_e = 0.05$
- Pilot/LMMSE, $P_e = 0.05$
- FASURA, $P_e = 0.05$

Tensor-Based Modulation by A. Decurninge, I. Land, & M. Guillaud
Orthogonal Pilots by M. J. Ahmadi & T. M. Duman
Pilot/MF by A. Fengler, O. Musa, P. Jung, & G. Caire
FASURA by M. Gkagkos, K. R. Narayanan, J.-F. Chamberland, and C. N. Georghiades
Pertinent References


Part V

Fast Fading Massive MIMO
Unsourced Random Access Channels
Signal model

- Signal received at time instant $t$ with slot $\ell$

\[ y(t, \ell) = \sum_{k=1}^{K} x_k(t, \ell) h_k(\ell) + z(t, \ell) \]

- Number of receive antennas $M \gg 1$
- Block fading – channel does not change within CCS slot
- Spatial correlation negligible – $h_k(\ell) \sim \mathcal{CN}(0, I_M)$
Multiple Measurement Vector – CS Interpretation

\[
\mathbf{A}(\ell), \frac{n}{L} \times 2^v \ell
\]

\[
\mathbf{Y}(\ell), \frac{n}{L} \times M
\]

\[
\Gamma(\ell) = \text{diag}(\mathbf{\gamma}(\ell)) \quad \mathbf{H}(\ell), 2^v \ell \times M
\]

\[
\mathbf{\gamma}(\ell) \in \{0, 1\}^{2^v \ell}
\]

\[
\|\mathbf{\gamma}(\ell)\|_0 = K
\]

- Received signal during slot \(\ell\): \(\mathbf{Y}(\ell) = \mathbf{A}(\ell)\Gamma(\ell)\mathbf{H}(\ell) + \mathbf{Z}(\ell)\)
- Column \(\mathbf{y}_i(\ell)\) of \(\mathbf{Y}(\ell)\) is the signal received at antenna \(i\) during slot \(\ell\)
- \(\mathbf{H}(\ell)\) has entries drawn i.i.d. from \(\mathcal{CN}(0, 1)\)
Divide-and-Conquer, again – Outer Tree Code
### Covariance-Based Estimation

#### Idea

Since channel vectors are Gaussian, columns $y_i(\ell)$ of $Y(\ell)$ are i.i.d. Gaussian $\mathcal{CN}(0, \Sigma_\ell)$

#### Computing covariance matrix

\[
\Sigma_\ell = \mathbb{E} [y_i(\ell)y_i(\ell)^H] = \frac{1}{M} \mathbb{E} [Y(\ell)Y(\ell)^H] \\
= \frac{1}{M} \mathbb{E} [(A(\ell)\Gamma(\ell)H(\ell) + Z(\ell)) (A(\ell)\Gamma(\ell)H(\ell) + Z(\ell))^H] \\
= \frac{1}{M} \left( A(\ell)\Gamma(\ell) \mathbb{E} \left[ H(\ell)H(\ell)^H \right] \Gamma(\ell)^H A(\ell)^H + \mathbb{E} \left[ Z(\ell)Z(\ell)^H \right] \right) \\
= A(\ell)\Gamma(\ell)A(\ell)^H + N_0 I_{n/L}
\]
Covariance-Based Estimation

Measurement $Y(\ell)$ is Gaussian with covariance

$$\Sigma_\ell = \frac{1}{M} \mathbb{E} \left[ Y(\ell)Y(\ell)^H \right] = A(\ell) \Gamma(\ell) A(\ell)^H + N_0 I$$

Use empirical average

- Let $\hat{\Sigma}_{Y(\ell)} = \frac{1}{M} Y(\ell)Y(\ell)^H$ be empirical covariance matrix
- Constrained ML estimate of $\gamma(\ell)$ given by

$$\gamma^*(\ell) = \arg \max_{\gamma(\ell) \in \mathbb{R}_{+}^{2^\ell}} \log p(Y(\ell)|\gamma(\ell))$$

$$= \arg \max_{\gamma(\ell) \in \mathbb{R}_{+}^{2^\ell}} \frac{1}{M} \sum_{i=1}^{M} \log p(y_i(\ell)|\gamma(\ell))$$

$$= \arg \min_{\gamma(\ell) \in \mathbb{R}_{+}^{2^\ell}} \left( \log |\Sigma_\ell| + \text{trace} \left( \Sigma^{-1}_\ell \hat{\Sigma}_{Y(\ell)} \right) \right)$$
Iterative Updates Based on Sherman-Morrison Identity

**Algorithm** Covariance Based Estimation via Coordinate Descent

**Inputs:** Sample covariance $\hat{\Sigma}_Y(\ell) = \frac{1}{M} Y(\ell) Y(\ell)^H$

**Initialize:** $\Sigma_\ell = N_0 I$, $\gamma(\ell) = 0$

**for** $i = 1, 2, \ldots$ **do**

**for** $k = 1, 2, \ldots, 2^v_\ell$ **do**

Set $d^* = \frac{a_k(\ell)^H \Sigma^{-1}_\ell (\hat{\Sigma}_Y(\ell) \Sigma^{-1}_\ell - I_n) a_k(\ell)}{(a_k(\ell)^H \Sigma^{-1}_\ell a_k(\ell))^2}$

Update $\gamma_k(\ell) \leftarrow \max\{\gamma_k(\ell) + d^*, 0\}$

Update $\Sigma^{-1}_\ell \leftarrow \Sigma^{-1}_\ell - \frac{d^* \Sigma^{-1}_\ell a_k(\ell) a_k(\ell)^H \Sigma^{-1}_\ell}{1 + d^* a_k(\ell)^H \Sigma^{-1}_\ell a_k(\ell)}$

**Output:** Estimate $\gamma(\ell)$

- Component-wise maximization of the log-likelihood cost function
- Guaranteed to converge to at least a local minimum
- Good empirical performance

Sherman-Morrison plus Tree Pruning

**Algorithm** Covariance Based Estimation via Coordinate Descent & SCLD

**Inputs:** Sample covariance $\hat{\Sigma}_Y(\ell) = \frac{1}{M} Y(\ell) Y(\ell)^H$

**Initialize:** $\Sigma_\ell = N_0 I$, $\gamma(\ell) = 0$

for $i = 1, 2, \ldots$ do

for $k \in S_\ell$ do

Set $d^* = \frac{a_k(\ell)^H \Sigma_\ell^{-1} (\hat{\Sigma}_Y(\ell) \Sigma_\ell^{-1} - I_n) a_k(\ell)}{(a_k(\ell)^H \Sigma_\ell^{-1} a_k(\ell))^2}$

Update $\gamma_k(\ell) \leftarrow \max\{\gamma_k(\ell) + d^*, 0\}$

Update $\Sigma_\ell^{-1} \leftarrow \Sigma_\ell^{-1} - \frac{d^* \Sigma_\ell^{-1} a_k(\ell) a_k(\ell)^H \Sigma_\ell^{-1}}{1 + d^* a_k(\ell)^H \Sigma_\ell^{-1} a_k(\ell)}$

**Output:** Estimate $\gamma(\ell)$

- Descent performed only over subset $S_\ell \subseteq \{2^{v_\ell}\}$ of columns in $A(\ell)$

- $S_\ell$ supplied by the outer tree decoder as side information

- Significant improvements in performance and complexity

Successful Cancellation List Decoding

- Active partial paths determine possible parity patterns
- Admissible indices for next slot determined by possible parities
- Inadmissible columns can be pruned before CS algorithm
Outer Tree Decoding – Dimensionality Reduction

Every surviving path produces parity pattern
Only fragments with these pattern can appear in subsequent slot
On average, there are $K(1 + \mathbb{E}[P_\ell])$ active paths
Outer Tree Decoding with Column Pruning

Possible indices

Original sensing matrix

Pruned matrix

- For $K$ small, width of sensing matrix is greatly reduced
- Actual sensing matrix is determined dynamically at run time
- Complexity of CS algorithm becomes much smaller
Parity allocation parameters, with \( w_\ell + p_\ell = 15 \),

\[
(p_1, p_2, \ldots, p_{10}) = (6, 8, 8, 8, 8, 8, 8, 13, 15)
\]

- Pruning is more pronounced at later stages
- Effective width of sensing matrix is greatly reduced
Performance Comparison

- Number of antennas reduced by 23% when $K = 100$
- Gains in computational complexity more pronounced when $K$ modest
- Reparametrization may offer additional gains
Pertinent References


Part VI

Providing Feedback:
Acknowledgments within URA
Feedback in Unsourced Random Access

- Existing URA schemes do not acknowledge reception of messages
- Acknowledgement (ACK) from BS is critical in certain applications
- Challenges of feedback in URA:
  - BS does not know which user sent each message
  - Large number of users

Section goal

Develop mechanism to acknowledge successful messages within URA

HashBeam: Feedback via Downlink Beamforming

Downlink beamforming in URA

- BS has channel estimate $h_i$ and message $m_i$ for every decoded UE
- UE $i$ and BS compute hash $a_i = f(m_i)$, $f : \{0, 1\}^B \rightarrow \mathbb{C}^L$
- $s_i = a_i \otimes h_i$ acts as identifier for UE $i$
- **Idea**: Leverage tensor $s_i$ to inform UE $i$
HashBeam: Feedback via Downlink Beamforming

\[ \begin{bmatrix} a_1 & a_2 & \ldots & a_K \end{bmatrix} \]
\[ \begin{bmatrix} h_1 & h_2 & \ldots & h_K \end{bmatrix} \]
\[ \otimes \otimes \ldots \otimes \]
\[ S = A \ast H \]

LMMSE beamforming

- Exploit uplink-downlink duality
- \[ W_{\text{LMMSE}}^H = (\alpha^2\sigma^2 I + S^H S)^{-1} S^H \in \mathbb{C}^{K \times LM} \]
- Base station transmits \( v = W_{\text{LMMSE}} 1 \in \mathbb{C}^{LM} \)
Decoding feedback at UE

- **UE** \(i\) receives \(r_i = [\langle h_i, v_1 \rangle + z_{i1}, \ldots, \langle h_i, v_L \rangle + z_{iL}] \in \mathbb{C}^L\)
- **UE** \(i\) decision statistic: \(\theta_i = \langle a_i, r_i \rangle\)
- Khatri-Rao \(S = A \ast H\) and \(v = S \left( \alpha^2 \sigma^2 I + S^H S \right)^{-1} 1\)

**Decision statistic**

\[
\theta_i = \langle a_i, r_i \rangle = \sum_j \left( a_{i,j}^* \langle h_i, v_j \rangle + a_{i,j}^* z_{i,j} \right) = \sum_j \left( \langle a_{i,j} h_i, v_j \rangle + a_{i,j}^* z_{i,j} \right) = \langle s_i, v \rangle + \langle a_i, z_i \rangle
\]

Compare \(\theta_i\) to quadratic decision boundary to detect ACK.
Structure of Decision Statistic $\theta$

$K = 10, \ M = 10, \ L = 7, \ \text{SNR} = 5 \ \text{dB}$

$K = 10, \ M = 5, \ L = 5, \ \text{SNR} = 15 \ \text{dB}$

Neyman-Pearson approach to quadratic decision region
- Fix $P_{MD} = 0.05$ and minimize $P_{FA}$
HashBeam: Feedback via Downlink Beamforming

Performance: required channel uses

- Number of channel uses $L$ scales as $O(K)$
- Adjust $L$ to adapt to any number of antennas $M$ or SNR
- Feedback is individualized - no common feedback signal
Additional discussion points

- Antennas from one to massive MIMO: In-between?
- Unrolling URA iterative algorithms and better channel models
- Heterogeneous classes of URA devices
- Incremental redundancy for URA: Quest for universality
- Over-the-air federated learning
- Cell-free URA and distributed iterative schemes
- Connection between URA and sketching in TCS

Questions?

Pertinent References


Universal Schemes and Incremental Redundancy

**Challenge**

- If device operates in isolation, it does not know total number of active devices $K$ nor slot count for current round.
- Packet count and sparse graph should have proper distribution at end of round.
- One way to fulfill requirement is for rolling message count to possess proper sparse distribution $\lambda_s(\cdot)$ at every time $s$.

**Can this be achieved?**

Hybrid ARQ for URA
Hybrid ARQ – Shifting from One Distribution to Another

1. Condition 1: Need enough probability mass to push over to neighbor
2. Condition 2: Can’t push probability mass past immediate neighbor
3. Conditions can be expressed mathematically in terms of first-order stochastic dominance

\[ X \preceq Y \text{ whenever } \Pr(X > m) \leq \Pr(Y > m) \quad \forall m \]

or, equivalently, cumulative distribution function (CDF) of \( X \) dominates CDF of \( Y \)
Cell-Free Unsourced Random Access

- Expected channel quality as a function of geographical location
- Received power decays, at least, quadratically with distance
Asynchronous Neighbor Discovery Using Coupled Compressive Sensing

Vamsi K. Amalladinne, Krishna R. Narayanan, Jean–Francois Chamberland, Dongning Guo

(Submitted on 2 Nov 2018)

The neighbor discovery paradigm finds wide application in Internet of Things networks, where the number of active devices is orders of magnitude smaller than the total device population. Designing low–complexity schemes for asynchronous neighbor discovery has recently gained significant attention from the research community. Concurrently, a divide–and–conquer framework, referred to as coupled compressive sensing, has been introduced for the synchronous massive random access channel. This work adapts this novel algorithm to the problem of asynchronous neighbor discovery with unknown transmission delays. Simulation results suggest that the proposed scheme requires much fewer transmissions to achieve a performance level akin to that of state–of–the–art techniques.

Subjects: Signal Processing (eess.SP); Information Theory (cs.IT)
Cite as: arXiv:1811.00687 [eess.SP]
(or arXiv:1811.00687v1 [eess.SP] for this version)

Building Robust Sensing Matrices

- Extending CCS framework with low sample complexity
- Addressing issues pertaining to asynchrony
- Context of neighbor discovery
Dealing with Jitter and Asynchrony

\[
y = A\tilde{x} + z \quad \text{with} \quad \|x\|_0 = K
\]

\[
A \in \mathbb{C}^{(n+T) \times 2^B} \quad \text{unknown due to unknown random delays}
\]

\[
\text{Max delay } T \text{ known to the decoder}
\]
Expanded Codebook through Sensing Matrix

Expanded Codebook $\mathbf{A}$

$(n + T) \times 2^B(T + 1)$ matrix

Accounts for all possible delays

$K$ out of $2^B(T + 1)$ sparse

Computational complexity of CS solvers: $O(\text{poly}(2^B(T + 1)))$
Sparsifying Collision & Data Fragmentation

CHIRRUP: a practical algorithm for unsourced multiple access

Robert Calderbank, Andrew Thompson

(Submitted on 2 Nov 2018)

Unsourced multiple access abstracts grantless simultaneous communication of a large number of devices (messages) each of which transmits (is transmitted) infrequently. It provides a model for machine-to-machine communication in the Internet of Things (IoT), including the special case of radio-frequency identification (RFID), as well as neighbor discovery in ad hoc wireless networks. This paper presents a fast algorithm for unsourced multiple access that scales to $2^{100}$ devices (arbitrary 100 bit messages). The primary building block is multiuser detection of binary chirps which are simply codewords in the second order Reed Muller code. The chirp detection algorithm originally presented by Howard et al. is enhanced and integrated into a peeling decoder designed for a patching and slotting framework. In terms of both energy per bit and number of transmitted messages, the proposed algorithm is within a factor of 2 of state of the art approaches. A significant advantage of our algorithm is its computational efficiency. We prove that the worst-case complexity of the basic chirp reconstruction algorithm is $O(nK(\log_2 n + K))$, where $n$ is the codeword length and $K$ is the number of active users, and we report computing times for our algorithm. Our performance and computing time results represent a benchmark against which other practical algorithms can be measured.

Subjects: Signal Processing (eess.SP)
Cite as: arXiv:1811.00879 [eess.SP]
(or arXiv:1811.00879v1 [eess.SP] for this version)

Submission history
From: Andrew Thompson [view email]
[v1] Fri, 2 Nov 2018 14:25:46 UTC (470 KB)

Which authors of this paper are endorsers? | Disable MathJax (What is MathJax?)

▶ Hadamard matrix based compressing scheme with CSS
▶ Ultra-low complexity decoding algorithm

Example: CHIRRUP

- Sensing matrix based on 2nd-order Reed-Muller functions,

\[ \phi_{R,b}(a) = \frac{(-1)^{wt(b)}}{\sqrt{2^m}} i^{(2b+Ra)^T a} \]

- Every column of form

\[
\begin{bmatrix}
\phi_{R,b}([0]_2) \\
\phi_{R,b}([1]_2) \\
\vdots \\
\phi_{R,b}([2^m - 1]_2)
\end{bmatrix}
\]

\([\cdot]_2\) is integer expressed in radix of 2

- Information encoded into \(R\) and \(b\)

- **Fast recovery:** Inner-products, Hardmard project onto Walsh basis, get \(R\) row column at a time, dechirp, Hadamard project to \(b\)
Thank You!

More available information at: https://engprojects.github.io/mMTC/

This material is based upon work supported, in part, by NSF under Grants CNS-2148354, CCF-2131106, and CCF-1619085

This material is also based upon work support, in part, by Qualcomm Technologies, Inc., through their University Relations Program