Unsourced Multiple Access (UMAC): Information Theory and Coding

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Presentation Outline and Learning Objectives

Unsourced random access

1. Motivation
2. CS and support recovery
3. Inference in large dimensions
4. Sparsifying collisions
5. Data fragmentation
6. Speading and tensors

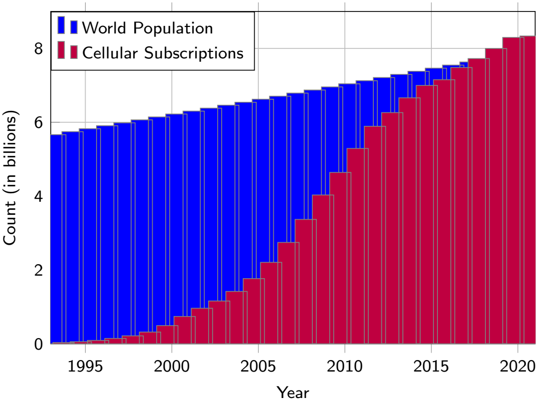
Tools for complexity

- Graph-based constructions
- Concatenated codes
- Data fragmentation
- Approximate message passing
- Fast transform methods
- Information propagation

Additional Resources

- PDF slides and source code
- `https://engprojects.github.io/mMTC/`
- `https://github.com/EngProjects/mMTC/` (branch: code)

World Population versus Subscribers



Fact 1: There are more active cellular subscriptions than there are people

World poputation: `https://www.statista.com/`; Cellular subscribers: `https://www.worldometers.info/`

Visual Acuity and Display Technology

Apple Super Retina HD

Visual resolution

Peak visual resolution of 20/20 human is

$$\frac{1}{\text{Visual Acuity}} = \frac{1}{20/20} \text{ min. of arc}$$
$$\approx 0.0167 \text{ degrees}$$

Sharp drops limit viewing angle to ± 20 degrees

Screen distance

The distance at which super retina HD display matches this resolution

$$\text{Distance} = \frac{1}{2} \cdot \frac{1}{458} \cdot \cot \frac{1}{120} = 1.876 \text{ in}$$

Fact 2: Display technology is reaching limit of visual acuity

Daily Use of Mobile Devices

Video and mobile statistics

- 63% of all US online traffic comes from smartphones and tablets – Stone Temple
- More than 70% of YouTube viewing happens on mobile devices – Comscore
- 65% of all digital media time is spent on mobile devices – Business2Community

Fact 3 Americans spend significant time on Mobile Devices. Average time spent on mobile phone in US exceeds 3h45m per day

– eMarketer

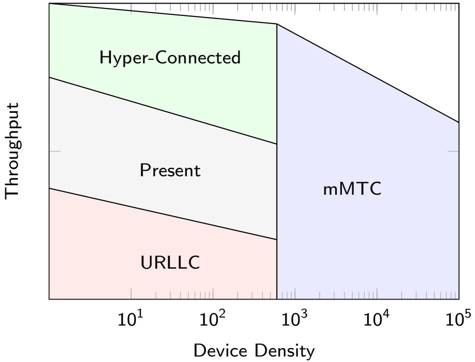
Quality of Experience

Current wireless landscape

- **Growth and Market Penetration**: Near saturation
  - Number of connected wireless devices exceeds world population
  - Almost every human who wants mobile phone has one (or more)
- **Screen Quality**: At limit of eye acuity
  - Screens are near boundary of visual resolution
  - Viewing distance is constrained by amplitude of accommodation
- **Content-Rich Apps**: Video watching & gaming are prevalent
  - On average, a person spends 4 hours on a mobile device per day
  - More videos are watched on phones than elsewhere

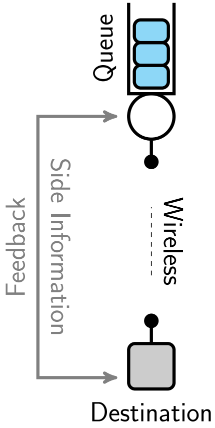
What's Next?

6G Envisioned Traffic Types



- Hyper-Connected Experience: XR, Hologram, Digital Replica
- Ultra-Reliable and Low Latency Communications (URLLC)
- massive Machine Type Communications (mMTC) – Uplink

An Evolving Wireless Landscape



Conventional systems

- Human operators, sustained connections
- Scheduling decisions based on channel quality & queue length
- Acquisition of side information amortized over long connections

Envisioned IoT environments

- Machine-to-machine communications
- Sporadic single transmissions from large number of devices
- Minute payloads

Information and Inference

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 51, NO. 2, FEBRUARY 2003 407

Decentralized Detection in Sensor Networks

Jean-François Chamberland, *Student Member, IEEE,* and Venugopal V. Veeravalli, *Senior Member, IEEE*

***Abstract*—In this paper, we investigate a binary decentralized detection problem in which a network of wireless sensors provides relevant information about the state of nature to a fusion center. Each sensor transmits its data over a multiple access channel. Upon reception of the information, the fusion center attempts to accurately reconstruct the state of nature. We consider the scenario where the sensor network is constrained by the capacity of the wireless channel over which the sensors are transmitting, and we study the structure of an optimal sensor configuration. For the problem of detecting deterministic signals in additive Gaussian noise, we show that having a set of identical binary sensors is asymptotically optimal, as the number of observations per sensor goes to infinity. Thus, the gain offered by having more sensors exceeds the benefits of getting detailed information from each sensor. A thorough analysis of the Gaussian case is presented along with some extensions to other observation distributions.**

***Index Terms*—Bayesian estimation, decentralized detection, sensor network, wireless sensors.**

problem have been studied in the past. Notably, the class of decentralized detection problems where each sensor must select one of D possible messages has received much attention. In this setting, which was originally introduced by Tenney and Sandell [1], the goal is to find what message should be sent by which sensor and when. See Tsitsiklis [2] and the references contained therein for an elaborate treatment of the decentralized detection problem. More recently, the problem of decentralized detection with correlated observations has also been addressed (see, e.g., [3] and [4]).

In essence, having each sensor select one of D possible messages upper bounds the amount of information available at the fusion center. Indeed, the quantity of information relayed to the fusion center by a network of L sensors, each sending one of D possible messages, does not exceed $L\lceil \log_2 D \rceil$ bits per unit time. In the standard decentralized problem formulation, the number of sensors L and the number of distinct messages D are

Payload design guideline

- Most of information for inference is contained in first few bits!

Information and Inference

A Telemetering System by Code Modulation —Δ-Σ Modulation*

H. INOSE†, MEMBER, IRE, Y. YASUDA†, AND J. MURAKAMI‡

Summary—A communication system by code modulation is described which incorporates an integration process in the original delta modulation system and is named delta-sigma modulation after its modulation mechanism. It has an advantage over delta modulation in dc level transmission and stability of performance, although both require essentially an equal bandwidth and complexity of circuitry. An experimental telemetering system employing delta-sigma modulation is also described.

the input signal before it enters the modulator so as to generate output pulses carrying the information corresponding to the amplitude of the input signal. The delta-sigma modulation (Δ-ΣM) system is a realization of this principle.

THE PRINCIPLE OF THE Δ-ΣM SYSTEM

Payload design guideline

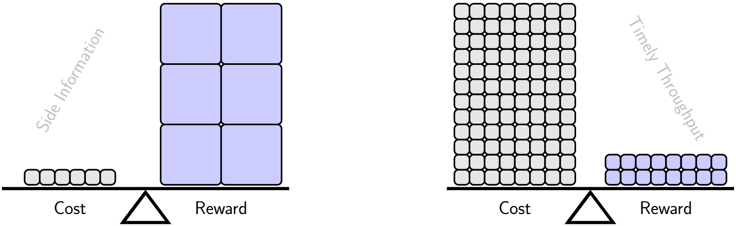
- Signals are tracked well using small, yet frequent updates
- Δ-Σ modulation

Losing the Connection

Emerging M2M traffic characteristics

- Device density – Massive versus small
- Connectivity profile – Sporadic versus sustained
- Packet payloads – Minuscule versus moderate-to-long

Anticipated traffic characteristics invalidate the acquisition-estimation-scheduling paradigm!



Revival of Uncoordinated Access

A new reality

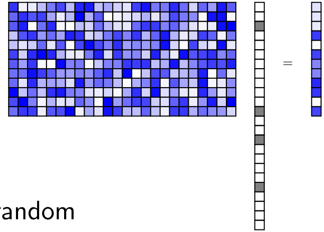
- Must address sporadic nature of machine-driven communications
- Transfer of small payloads without ability to amortize cost of acquiring channel and buffer states over long connections
- Preclude use of opportunistic scheduling

Communication and identity

When number of devices is massive, with only subset of them active, problem of allocating resources (e.g., codebook, subcarriers, signature sequences) to every user as to manage interference becomes complex

Uncoordinated, Unsourced MAC

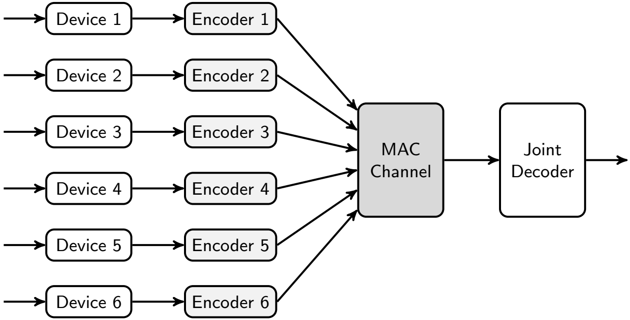
Unsourced and Uncoordinated Random Access



Section Objectives

1. Review connection between unsourced random access and compressed sensing
2. Understand challenges with URA and sparse recovery in exceedingly large dimensional spaces
3. Introduce candidate design strategies to address this research problem

Uncoordinated Multiple Access Channel (MAC)

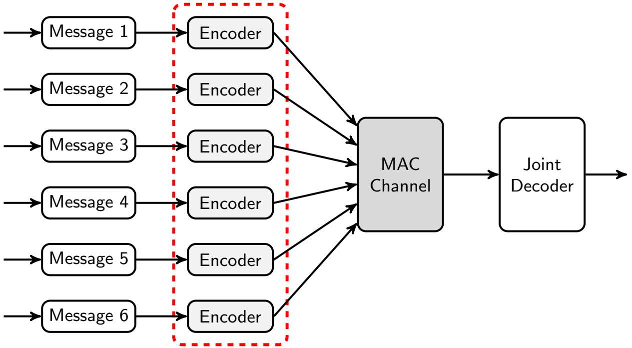


LoRa-inspired parameters

- K active devices out of K_{tot} $K \in [25 : 300]$
- Each device has B-bit message, B is small ≈ 128
- n channel uses available, $n \approx 30,000$

M. Berioli, G. Cocco, G. Liva and A. Munari. *Modern Random Access Protocols.* Foundations and Trends in Networking, 2016

Uncoordinated and Unsourced MAC



No personalized feedback

- All devices use same codebook
- No explicit knowledge of identities
- Decoder returns unordered list

Mathematical model

$$\mathbf{y} = \sum_i \mathbf{x}_i + \mathbf{z}$$

where $\mathbf{x}_i$ depends on message

Gaussian Random Codes & Performance Bounds

A perspective on massive random-access

Yury Polyanskiy

***Abstract*—This paper discusses the contemporary problem of providing multiple-access (MAC) to a massive number of uncoordinated users. First, we define a random-access code for K_a-user Gaussian MAC to be a collection of norm-constrained vectors such that the noisy sum of any K_a of them can be decoded with a given (suitably defined) probability of error. An achievability bound for such codes is proposed and compared against popular practical solutions: ALOHA, coded slotted ALOHA, CDMA, and treating interference as noise. It is found out that as the number of users increases existing solutions become vastly energy-inefficient.**

MAC [11], [12]). Already 30 years ago R. Gallager [13] called for "a coding technology that is applicable for a large set of transmitters of which a small, but variable, subset simultaneously use the channel." It appears (to this author) that this call has not been completely answered still. One reason for this could be that the models in each of three categories are different and thus solutions are not directly comparable. Our first goal, thus, is to define a notion of random-access code that would appeal to all three communities. This we do next.

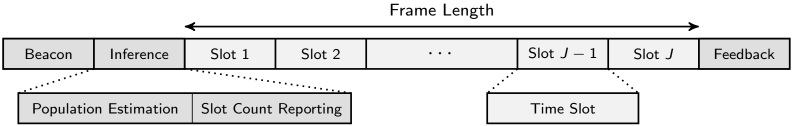
Theorem: Fix $P' < P$. There exists an (M, n, ϵ) random-access code for the K-user GMAC satisfying power-constraint P and

$$\epsilon \leq \sum_{t=1}^{K} \frac{t}{K} \min(p_t, q_t) + p_0,$$

where constants p_0, p_t, and q_t are complicated

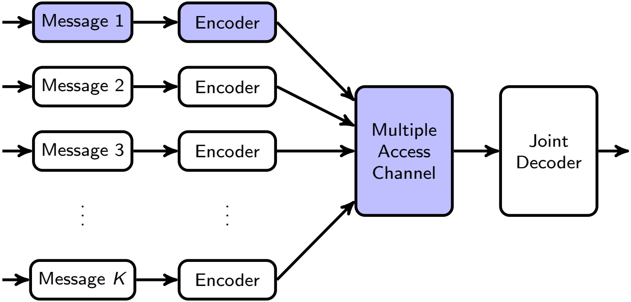
Uncoordinated MAC Frame Structure

- K active devices out of many, many devices
- Framework of gathering channel and queue states does not apply



- Beacon employed for coarse synchronization
- Same set of devices transmit within frame
- Frame may or may not have slots
- Each device may or may not use every slot

URA Framework and Sparse Recovery

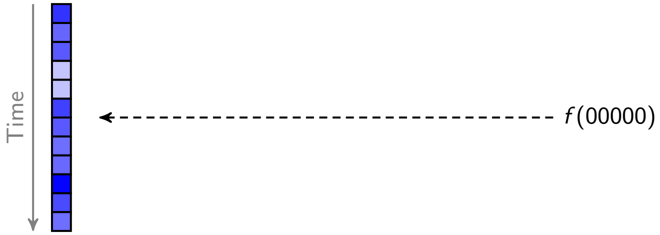


Characteristics of URA framework

- Every device employs same codebook $f : \{0,1\}^B \to \mathbb{R}^n$
- Decoder must produce unordered list of messages

URA – Compiling Signal Dictionary

Message 00000 maps to column



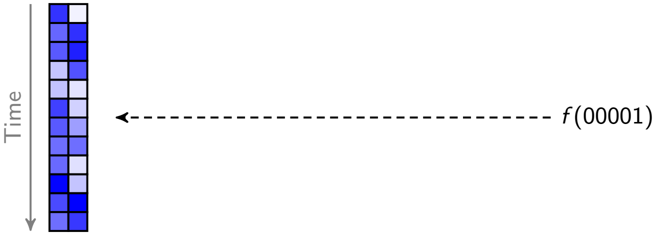
Message Encoding:

$$f : \{0, 1\}^B \mapsto \mathbb{R}^n$$

$$f(\text{binary message}) = \text{signal}$$

URA – Compiling Signal Dictionary

Message 00001 maps to column

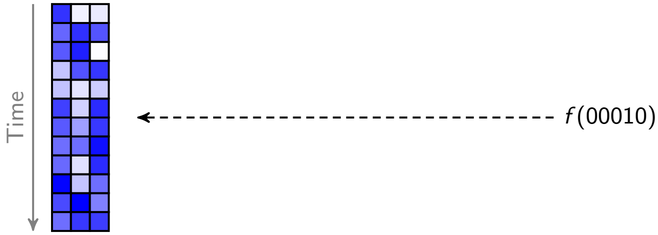


Message Encoding:

$$f : \{0,1\}^B \mapsto \mathbb{R}^n$$
$$f(\text{binary message}) = \text{signal}$$

URA – Compiling Signal Dictionary

Message 00010 maps to column

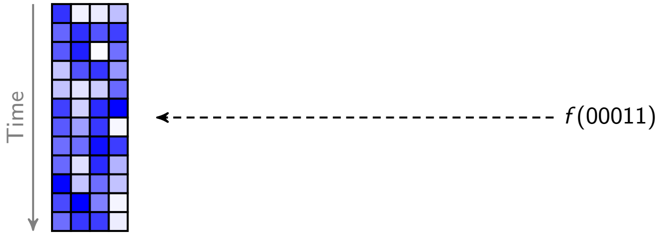


Message Encoding:

$$f : \{0,1\}^B \mapsto \mathbb{R}^n$$
$$f(\text{binary message}) = \text{signal}$$

URA – Compiling Signal Dictionary

Message 00011 maps to column

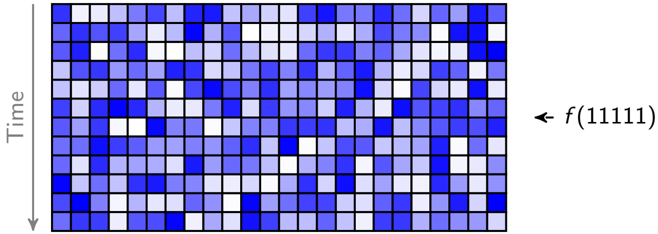


Message Encoding:

$$f : \{0,1\}^B \mapsto \mathbb{R}^n$$
$$f(\text{binary message}) = \text{signal}$$

URA – Compiling Signal Dictionary

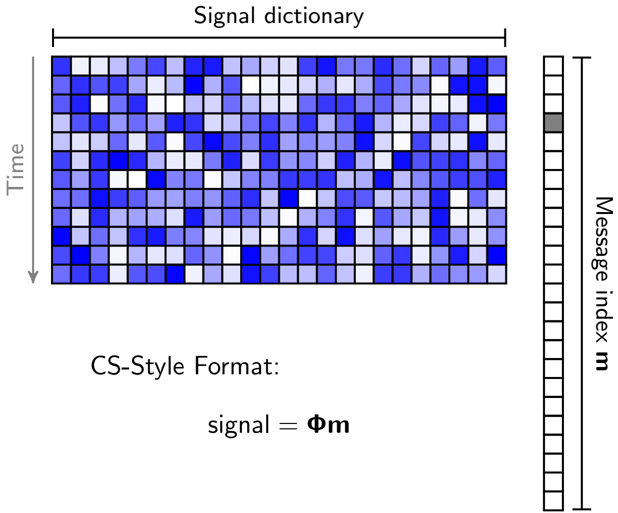
Message 11111 maps to column



Message Encoding:

$$f : \{0,1\}^B \mapsto \mathbb{R}^n$$
$$f(\text{binary message}) = \text{signal}$$

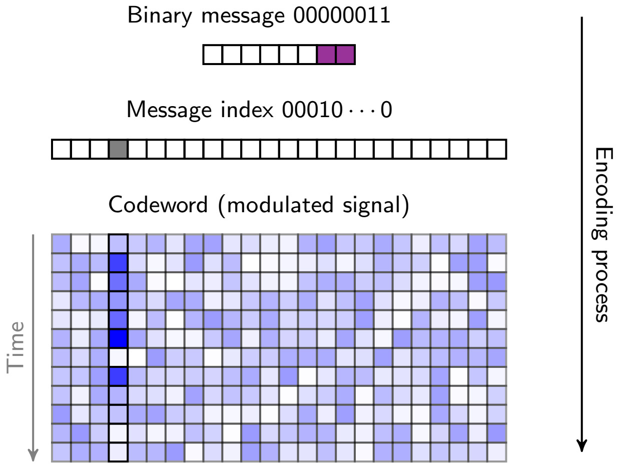
URA – Index Representation



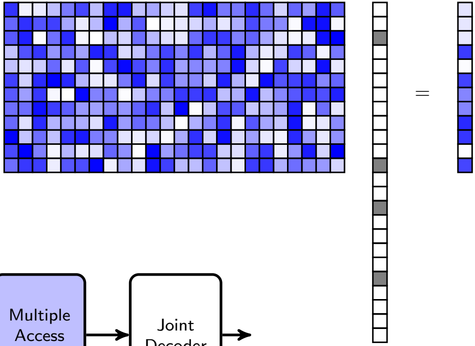
CS-Style Format:

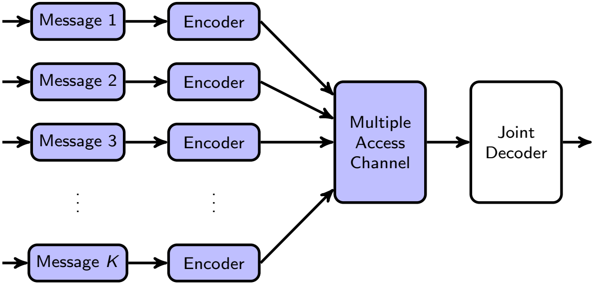
$$\text{signal} = \mathbf{\Phi m}$$

URA – Index Representation



Unsourced Random Access – CS Analogy





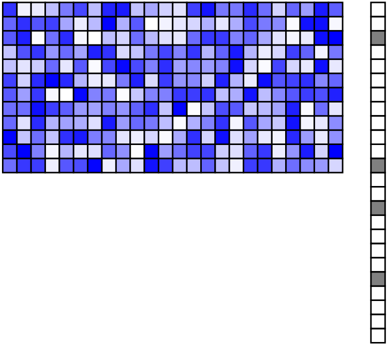
Abstract CS Challenge

Problem setting

- Noisy compressed sensing

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} + \mathbf{z}$$

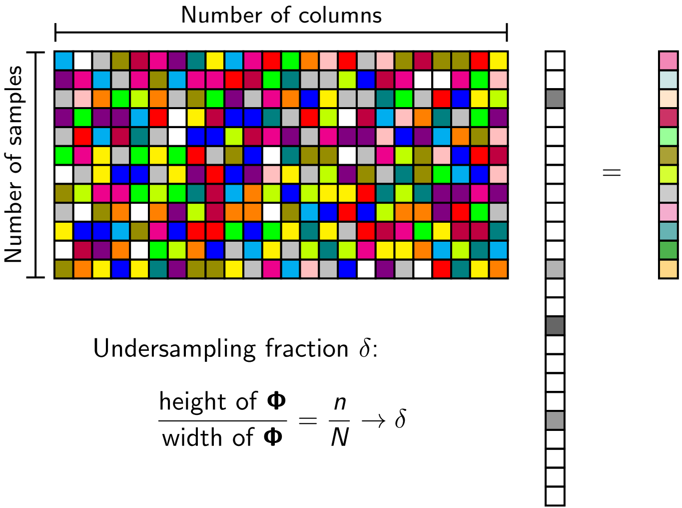
  where $\mathbf{s}$ is K sparse
- $\mathbf{s}$ has non-negative integer entries
- $\mathbf{\Phi}.\mathtt{shape} \approx 32,768 \times 2^{128}$
- $\mathbf{z}$ is additive Gaussian noise



Practical issues

- Width of sensing matrix is huge
- Existing CS solvers will not execute at that scale

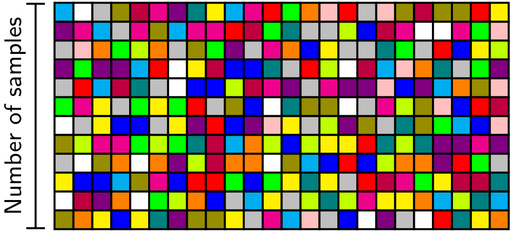
Compressed Sensing – Undersampling

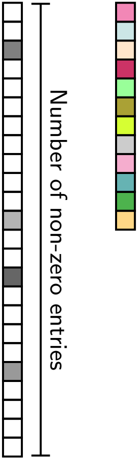


Undersampling fraction δ:

$$\frac{\text{height of } \mathbf{\Phi}}{\text{width of } \mathbf{\Phi}} = \frac{n}{N} \to \delta$$

Compressed Sensing – Sparsity

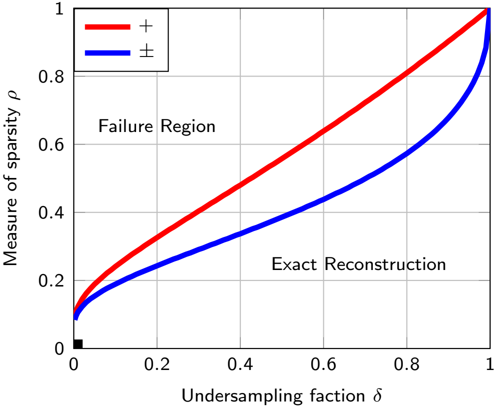




Measure of sparsity ρ:

$$\frac{\text{Signal complexity } \|\mathbf{s}\|_0}{\text{height of } \boldsymbol{\Phi}} = \frac{K}{n} \to \rho$$

Compressed Sensing – Phase Transitions



- Undersampling fraction

$$\delta = \frac{n}{N} = \frac{32,768}{2^{128}} = 2^{-113}$$

- Measure of sparsity

$$\rho = \frac{K}{n} = \frac{256}{32,768} = 2^{-7}$$

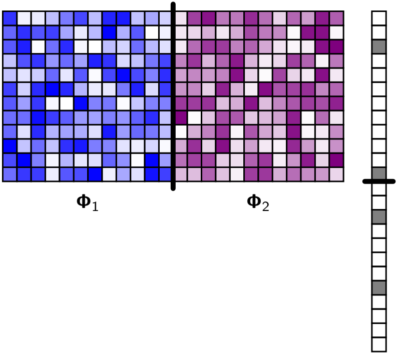
Classical Coding Techniques

Multi-user coding

- Matrix becomes codebooks

$$\mathbf{y} = \mathbf{\Phi}_1 \mathbf{s}_1 + \mathbf{\Phi}_2 \mathbf{s}_2 + \mathbf{z}$$

- Device picks code based on bits
- Well-studied for single user
- Fast decoding for large dictionary



Drawbacks

- Low complexity joint multi-user decoders are not available
- Devices may collide within codebook selection

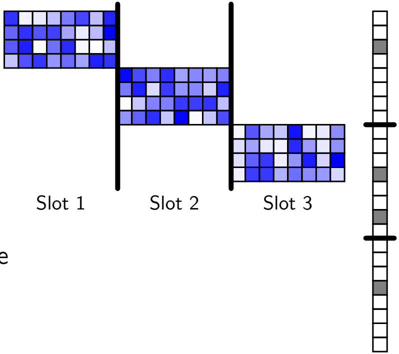
Time-Division Unsourced Random Access

Slot partitioning

- Observations become

$$\mathbf{y}_\ell = \mathbf{\Phi}_\ell \mathbf{s}_\ell + \mathbf{z}_\ell$$

  where ℓ is slot label
- Device gets slot based on message
- Channel uses divided among slots

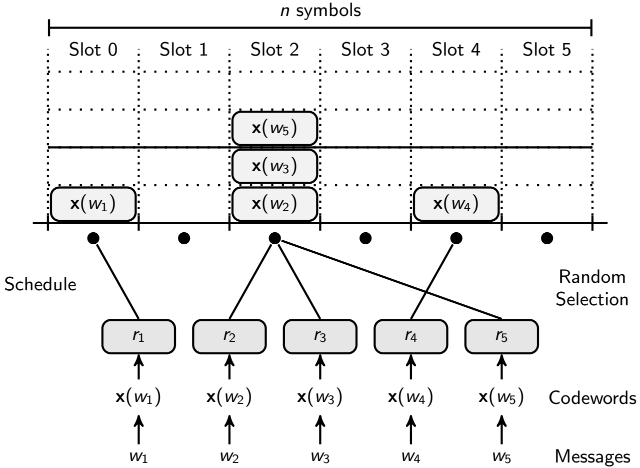


Drawbacks

- Matrices remain wide 2^{128} / # slots
- Devices assigned randomly within slots

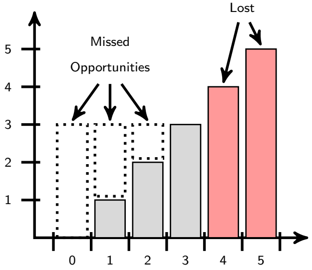
Quest for Low-Complexity Unsourced MAC

Sparsifying collision via stochastic binning

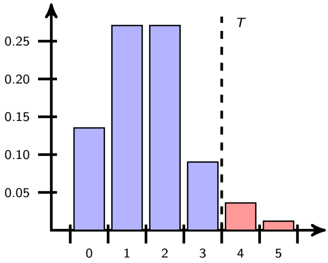


O. Ordentlich and Y. Polyanskiy. *Low complexity schemes for the random access Gaussian channel*. ISIT, 2017.

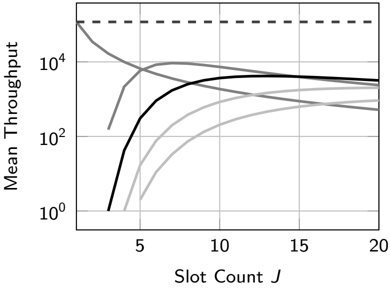
Caveat – The Poisson Wall



Sum Reward



Count Distribution

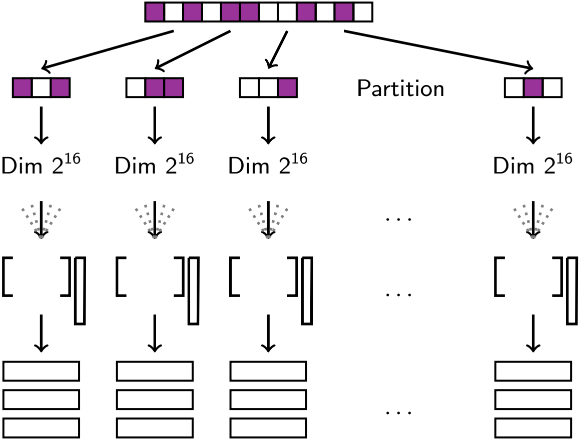


Effects of decoding threshold

- More slots reduces parameter of Poisson/binomial distribution
- More slots reduces bit count per decoded slot

$\sum_{k=0}^{T} \frac{N}{J} \frac{k}{T} \log_2 (1 + JT \cdot \mathrm{SNR}) \, \mathrm{pmf}(k)$

Data Fragmentation



Drawbacks

- Unordered lists of fragments
- Need to perform disambiguation

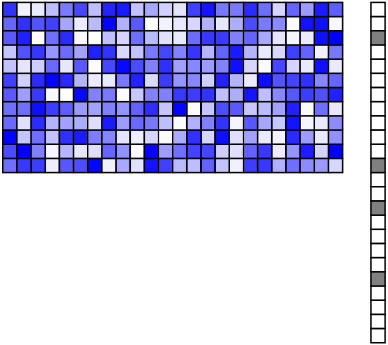
Section Summary

Problem formulation

- Noisy compressed sensing

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} + \mathbf{z}$$

- URA is noisy support recovery
- Full control over $\mathbf{\Phi}$
- Width of sensing matrix is huge
- Uncoordinated access produces stochastic binning



Possible URA design strategies

- Sparsifying collisions
- Advanced coding and spreading
- Data fragmentation

Pertinent References

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Part I

Sparsifying Collisions: Graph-Based Techniques and Concatenated Coding

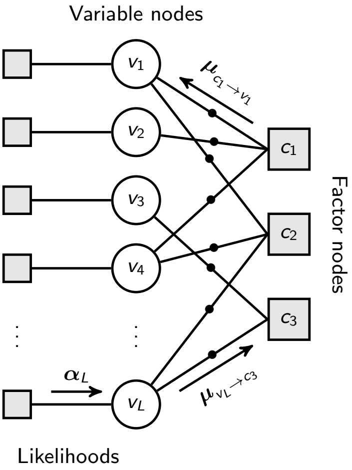
Sparsifying Collisions

Graphs

- Quest for low complexity URA (mMTC) schemes
- Draw inspiration from graphical models
- Belief propagation
- Survey past successes 3 ways
  1. LDPC/LDGM codes
  2. Compressed sensing
  3. Random access

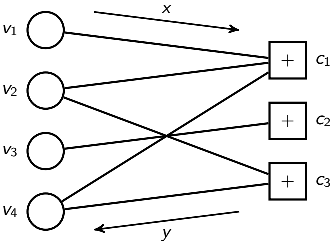
Section goal

Gain ability to create graph-based URA schemes

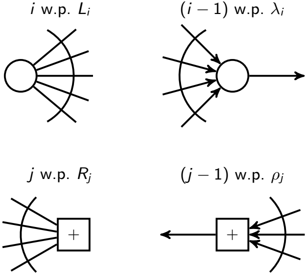


Sparse Graph: Tools from Iterative Decoding

- L_i variable dist. from node
- λ_i variable dist. from edge

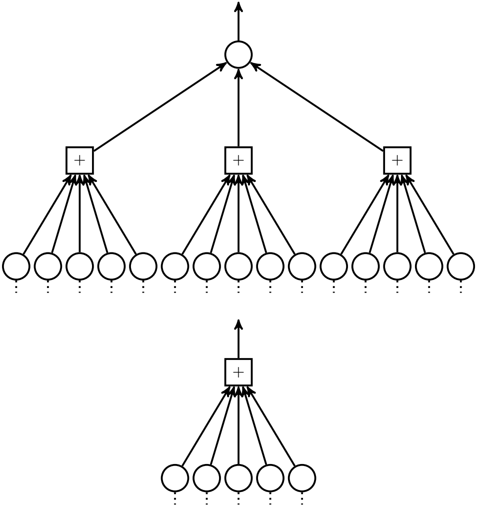


- R_j check dist. from node
- ρ_j check dist. from edge



V. Zyablov, and M. Pinsker. *Decoding complexity of low-density codes for transmission in a channel with erasures*. Problemy Peredachi Informatsii, 1974.

Sparse Graph: Computation Tree



Standard tricks

- Unravel bipartite graph
- Graph needs to be locally tree-like
- Focus on outgoing messages
- Analyze over random code ensemble

M. Luby, M. Mitzenmacher, A. Shokrollahi, and D. Spielman. *Efficient erasure correcting codes*. IEEE Trans. Information Theory, 2001.

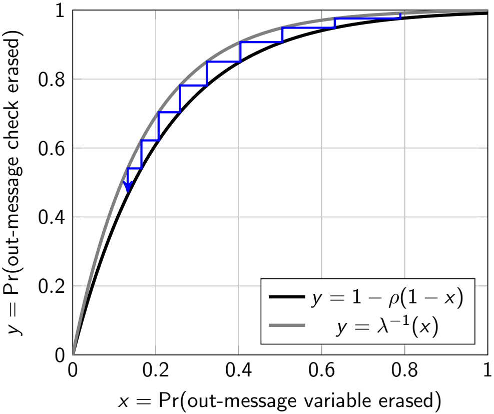
Sparse Graph: Analyzing Iterative Decoding

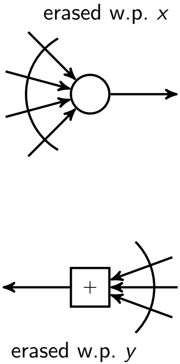
- x: Prob. outgoing message from variable node erased
- y: Prob. outgoing message from check node erased

- Outgoing variable message erased if all incoming check messages are erased
$$x = \mathrm{E}\left[y^{i-1}\right] = \lambda(y)$$
- Outgoing check message erased if any incoming variable message is erased
$$y = \mathrm{E}\left[1-(1-x)^{j-1}\right] = 1-\rho(1-x)$$

T. Richardson, and R. Urbanke. *Modern Coding Rheory*. Cambridge University Press, 2008.

Sparse Graph: EXIT Chart

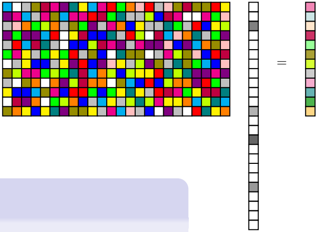




Extrinsic Information Transfer (EXIT) chart

$$y = 1 - \rho(1-x) \qquad\qquad x = \lambda(y) \quad \text{(flipped)}$$

Sparse Graph Code Based Compressed Sensing



Support recovery problem

$$\mathbf{y} = \mathbf{\Phi x} + \mathbf{z}$$

- Sensing matrix $\mathbf{\Phi}$ is $n \times N$
- Variable $\mathbf{z}$ is additive noise
- Recover $\text{supp}(\mathbf{x}) = \{i : \mathbf{x}_i \neq 0\}$

Objective

Devise scheme with minimal number of measurements n and minimal decoding complexity such that $\Pr(\text{failure}) \to 0$ as N (and K) $\to \infty$

Support Recovery – Fundamental Limit

Optimal order for support recovery

- In the sub-linear sparsity regime, $K = o(N)$, necessary and sufficient conditions are shown to be:

$$C_1 K \log\left(\frac{N}{K}\right) < n < C_2 K \log\left(\frac{N}{K}\right)$$

- In the linear sparsity regime, $K = \alpha N$, it was shown that $n = \Theta(N)$ measurements are sufficient for asymptotically reliable recovery.
- Minimum value $\mathbf{x}_{min}$ has to be bounded away from zero

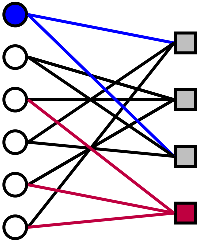
M. Wainwright. *Theoretic limits of support recovery*. IEEE Trans. Information Theory, 2009.
S. Aeron, V. Saligrama, and M. Zhao. *Information-theoretic bounds for compressed sensing*. IEEE Trans. Information Theory, 2010.

Sparse Graph Code Based Compressed Sensing

- Sparse-graph codes peeling decoder framework for CS
- Sample and measurement complexities of order $O(K \log N)$ for noisy setting[1]
- Complexities of $O(K \log N/K)$ for noisy setting[2]

Biadjacency matrix

$$\mathbf{A} = \begin{bmatrix} \color{blue}{1} & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ \color{blue}{1} & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \color{purple}{1} & 0 & \color{purple}{1} & \color{purple}{1} \end{bmatrix}$$



[1] X. Li, D. Yin, R. Pedarsani, S. Pawar, K. Ramchandran, *Sub-linear compressed sensing for support recovery using sparse-graph codes*. IEEE Trans. Information Theory, 2019.

[2] A. Vem, N. Thenkarai-Janakiraman, K. R. Narayanan. *Sub-linear time compressed sensing for support recovery using left and right regular sparse-graph codes*. ITW, 2016.

Tensoring Construction

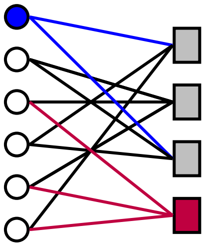
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} +1 & -1 & -1 \\ -1 & +1 & -1 \end{bmatrix}$$

Sensing matrix is tensor-inspired product

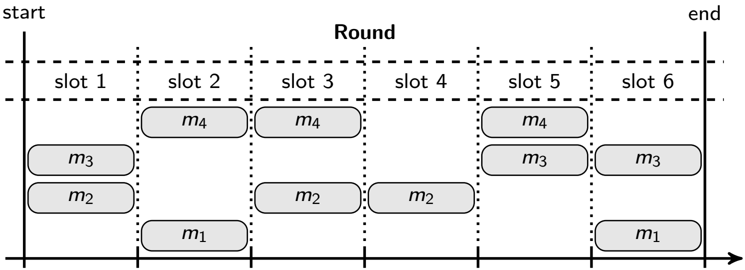
$$\mathbf{A} \boxplus \mathbf{S} = \left[\begin{array}{rrrrrr} +1 & 0 & 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & +1 & 0 & -1 \\ \hline 0 & +1 & -1 & 0 & -1 & 0 \\ 0 & -1 & +1 & 0 & -1 & 0 \\ \hline +1 & -1 & 0 & -1 & 0 & 0 \\ -1 & +1 & 0 & -1 & 0 & 0 \\ \hline 0 & 0 & +1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & +1 & -1 \end{array}\right]$$

Decoding checks



Design amenable to peeling decoding when $\mathbf{x}$ is real vector

Random Access with Twist



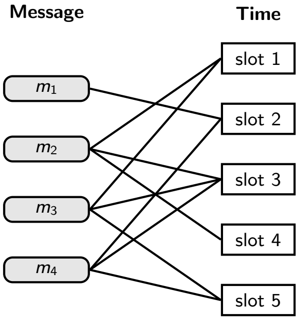
System model

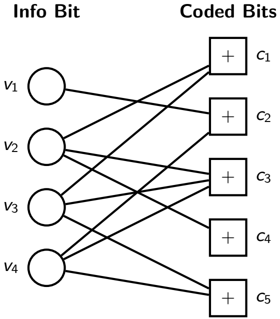
- K **uncoordinated** devices, each with 1 packet to send
- Time is **slotted**; transmissions occur within slots
- Receiver knows full schedule, collection of packets in every slot
- Successive interference cancellation

E. Casini, R. De Gaudenzi, and O. Del Rio Herrero. *Contention resolution diversity slotted ALOHA (CRDSA): An enhanced random access scheme for satellite access packet networks*. IEEE Trans. Wireless Communications, 2007.

Graphical Representation

- Tanner graph representation for LDGM transmission scheme
- Info nodes $\leftrightarrow$ packets; Coded nodes $\leftrightarrow$ received signals
- Message-passing (SIC) – **peeling decoder** for erasure channel



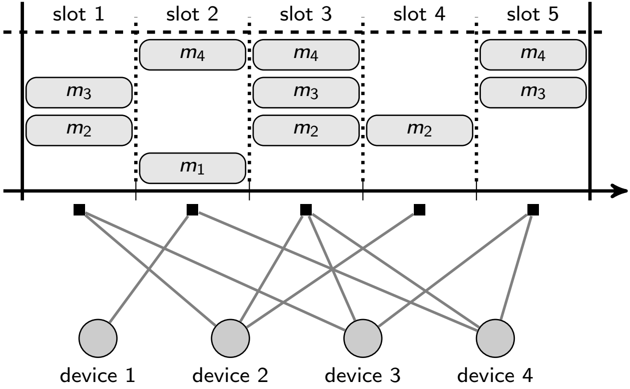


G. Liva. *Graph-based analysis and optimization of contention resolution diversity slotted ALOHA*. IEEE Trans. Communications, 2011.

E. Paolini, G. Liva, and M. Chiani. *Coded slotted ALOHA: A graph-based method for uncoordinated multiple access*. IEEE Trans. Information Theory, 2015.

Decoder – Peeling Algorithm

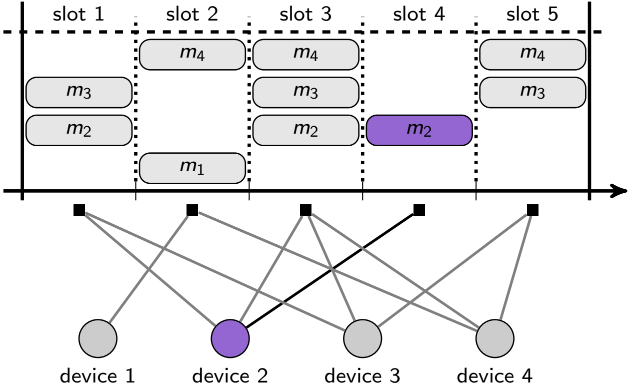
Joint decoding via successive interference cancellation



Instance of Random Access

Decoder – Peeling Algorithm

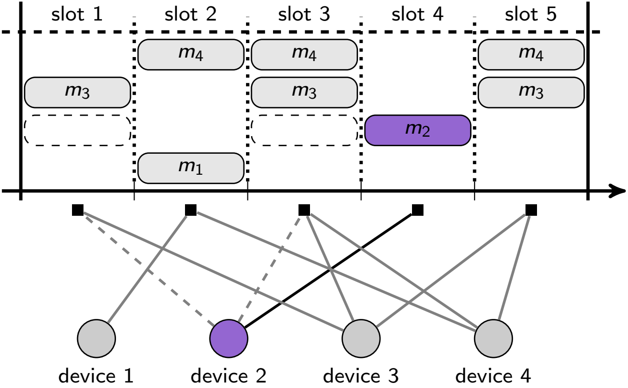
Joint decoding via successive interference cancellation



Step 1

Decoder – Peeling Algorithm

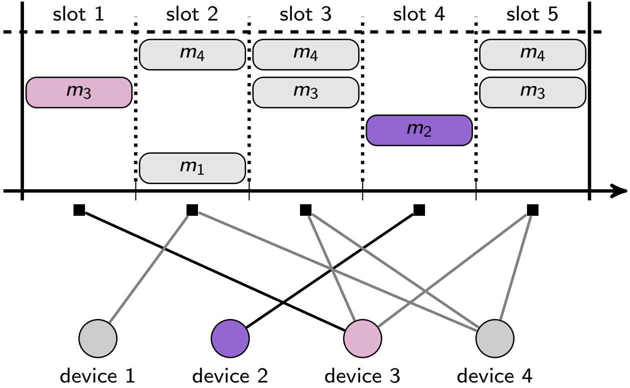
Joint decoding via successive interference cancellation



Step 1

Decoder – Peeling Algorithm

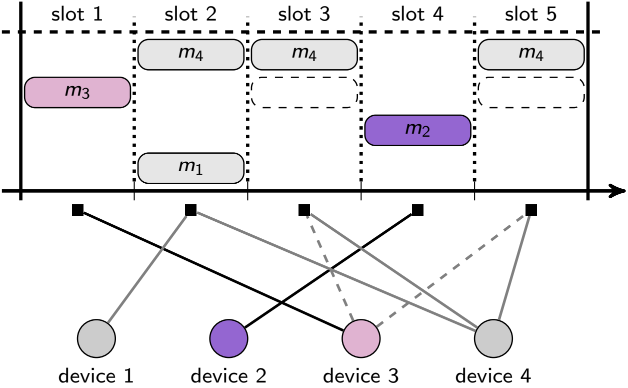
Joint decoding via successive interference cancellation



Step 2

Decoder – Peeling Algorithm

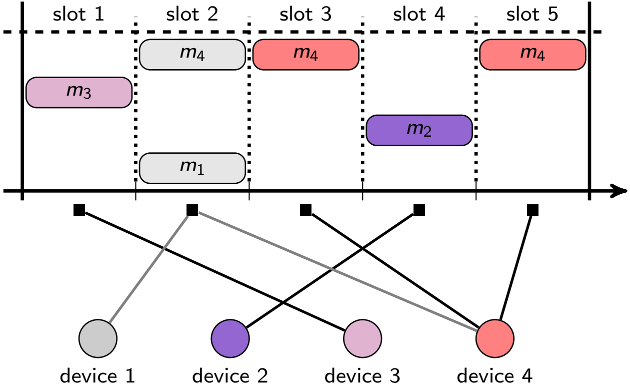
Joint decoding via successive interference cancellation



Step 2

Decoder – Peeling Algorithm

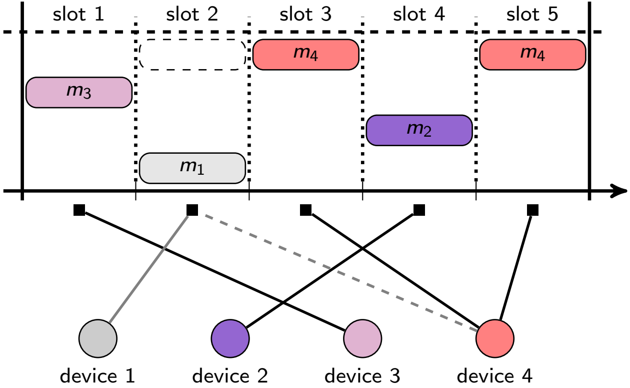
Joint decoding via successive interference cancellation



Step 3

Decoder – Peeling Algorithm

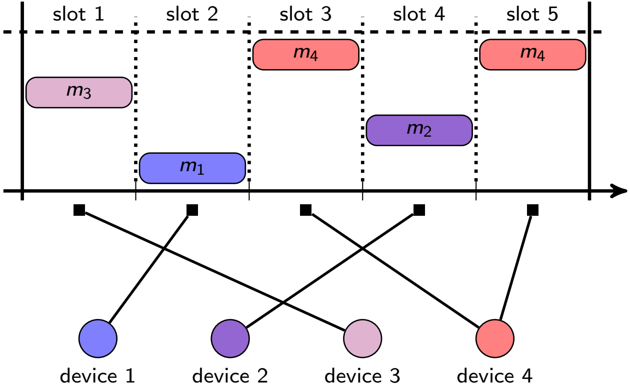
Joint decoding via successive interference cancellation



Step 3

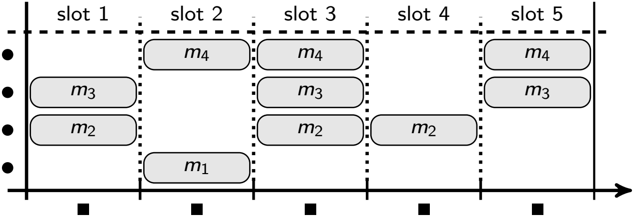
Decoder – Peeling Algorithm

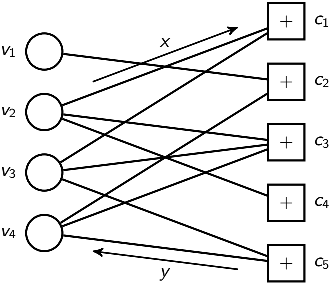
Joint decoding via successive interference cancellation

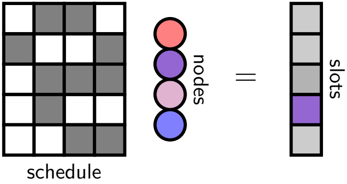


Step 4

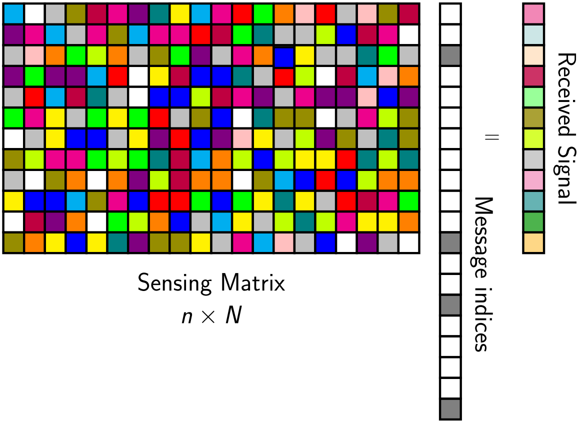
Representations: Schedule, Tanner Graph, Compressed





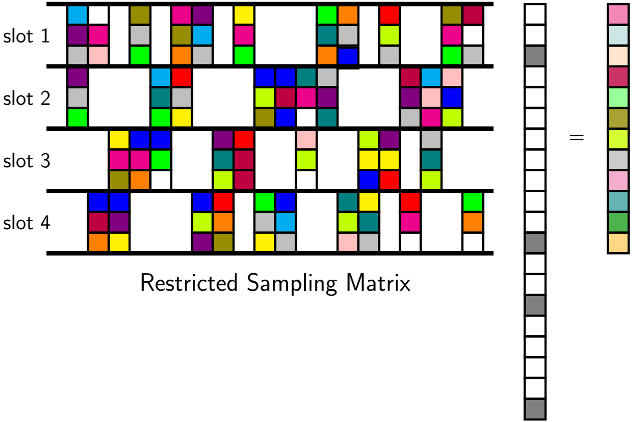


Sparse Graph – Back to Unsourced Random Access



- $N = 2^{128}$ columns
- $K \approx 100$ active devices
- Non-negative coefficients
- $n \approx 30,000$ measurements
- Complexity $\mathcal{O}(K \log N)$
- Support recovery

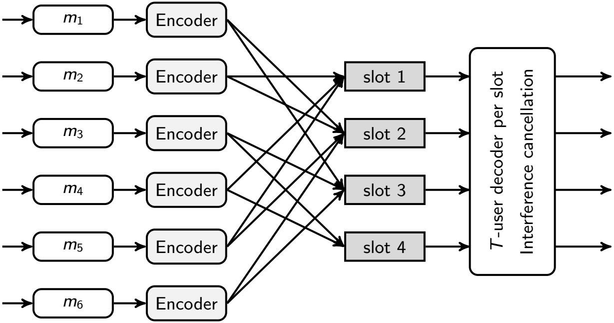
URA – A Quest for Low Complexity



Restricted Sampling Matrix

- Partition into J slots
- $\tilde{n} = n/J$ channel uses
- Aim is T-user adder channel
- Admits graphical representation

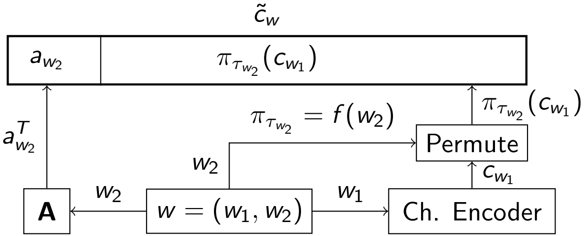
URA – Proposed Scheme



- Schedule selected based on **message**
- Devices can transmit in multiple sub-blocks
- Scheme facilitates successive interference cancelation

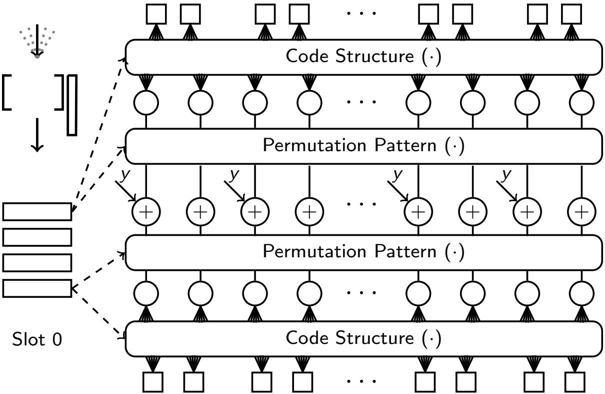
A. Vem, K. R. Narayanan, JFC, and J. Cheng. *A user-independent successive interference cancellation based coding scheme for the unsourced random access Gaussian channel*. IEEE Trans. Communications, 2019.

What Really Happens within Slot?



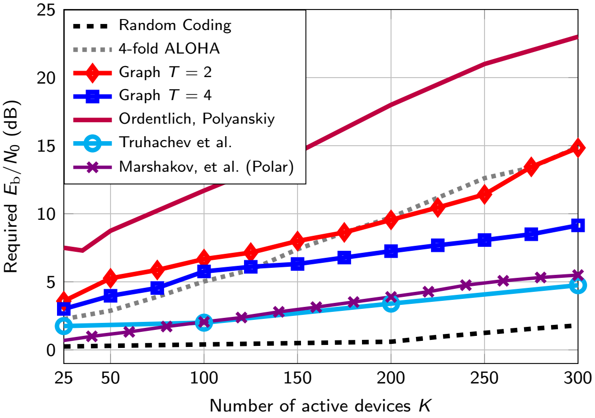
- Message is partitioned into two parts $w = (w_1, w_2)$
- Every device uses identical codebook built from LDPC-type codes tailored to T-user real-adder channel
- w_1 is encoded with a spatially-coupled LDPC code and then permuted based on w_2
- w_2 is compressed via CS matrix $\mathbf{A}$ and recovered through non-negative ℓ_1-regularized LASSO

Sparse Unsourced Random Access



- Compressed sensing preamble with information bits
- Sparse graph-based random access scheme conducive to joint decoding

Sparsifying Collision with Graph-Based Techniques



- Minimum $E_{\rm b}/N_0$ required as function of # of devices
- For $T = 2, 4$ and 4-fold ALOHA, prob. of decoding every slot ≥ 0.99
- Prob. recovered messages ≥ 0.96 given T-user decoding successful

Background and Pertinent References

- G. Liva, Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA. *IEEE Trans. Communications*, 2011.
- E. Paolini, G. Liva and M. Chiani. Coded Slotted ALOHA: A Graph-Based Method for Uncoordinated Multiple Access. *IEEE Trans. Information Theory*, 2015.
- O. Ordentlich and Y. Polyanskiy. Low complexity schemes for the random access Gaussian channel. In *IEEE International Symposium on Information Theory (ISIT)*, 2017.
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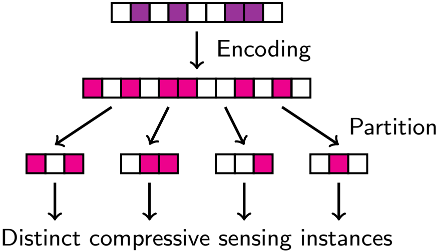
- A. Glebov, N. Matveev, K. Andreev, A. Frolov and A. Turlikov. Achievability Bounds for T-Fold Irregular Repetition Slotted ALOHA Scheme in the Gaussian MAC. In *IEEE Wireless Communications and Networking Conference (WCNC)*, 2019.
- G. Kasper Facenda and D. Silva. Efficient Scheduling for the Massive Random Access Gaussian Channel. *IEEE Trans. Wireless Communications*, 2020.
- D. Truhachev, M. Bashir, A. Karami, E. Nassaji. Low-complexity coding and spreading for unsourced random access. *IEEE Comm. Letters*, 2021.
- E. Nassaji, M. Bashir, D. Truhachev. Unsourced Random Access Over Fading Channels via Data Repetition, Permutation, and Scrambling. *IEEE Trans. Communications*, 2021.
- C. R. Srivatsa and C. R. Murthy. User activity detection for irregular repetition slotted ALOHA based MMTC. In *IEEE Transactions on Signal Processing*, 2022.

Part II

Data Fragmentation: A Divide-and-Conquer Approach to Compressed Sensing

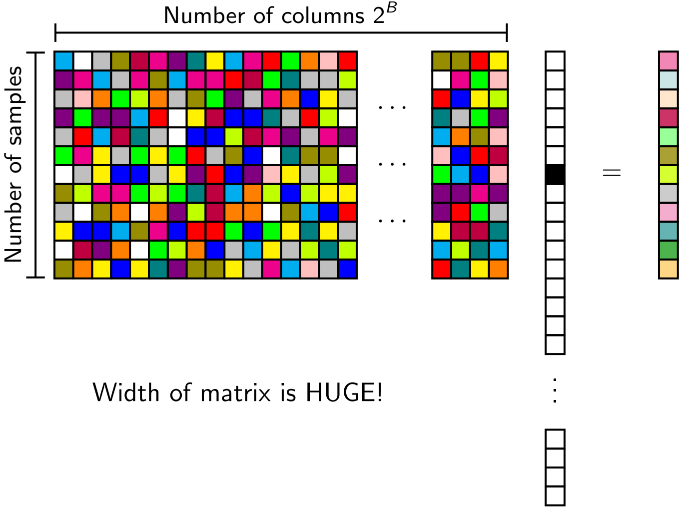
CCS – Fragmentation with Disambiguation



Stitching through outer code

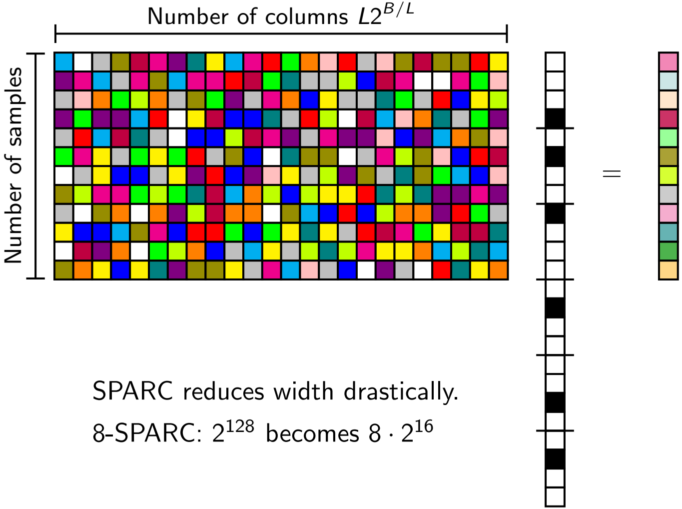
- Split problem into sub-components suitable for CS framework
- Get lists of sub-packets, one list for every slot
- Stitch pieces of one packet together using error correction

CCS – Sensing Matrix, Single-User Indexing



Width of matrix is HUGE!

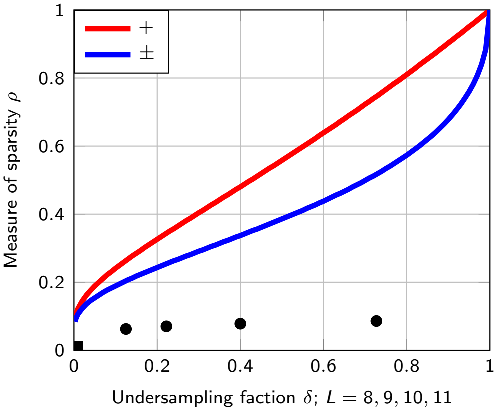
CCS – Sensing Matrix, Single-User SPARC



SPARC reduces width drastically.

8-SPARC: 2^{128} becomes $8 \cdot 2^{16}$

Drastic Reduction in Matrix Width



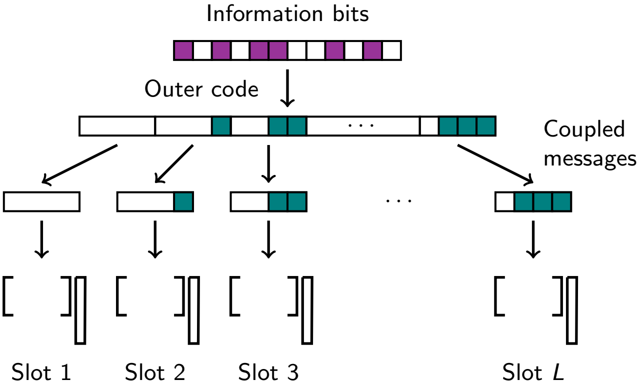
- Undersampling fraction

$$\delta = \frac{n}{N} = \frac{32,768}{L \cdot 2^{\lceil 128/L \rceil}}$$

- Measure of sparsity

$$\rho = \frac{K}{n} = \frac{L \cdot 256}{32,768} = \frac{L}{2^7}$$

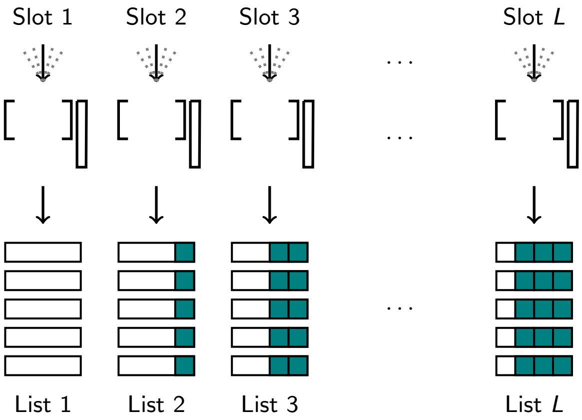
Coded Compressive Sensing – Device Perspective



- Collection of L CS matrices and 1-sparse vectors
- Each CS generated signal is sent in specific time slot

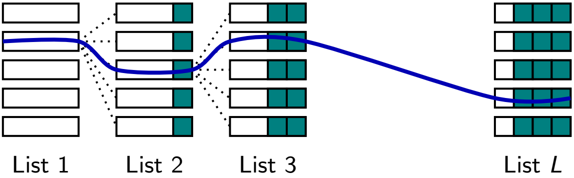
V. K. Amalladinne, JFC, and K. R. Narayanan. *A coded compressed sensing scheme for unsourced multiple access*. IEEE Trans. Information Theory, 2020.

Coded Compressive Sensing – Multiple Access



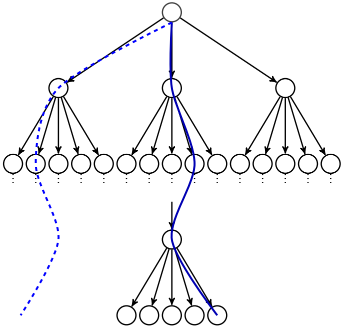
- L instances of CS problem, each solved with non-negative LS
- Produces L lists of K decoded sub-packets (with parity)
- Must piece sub-packets together using tree decoder

Coded Compressive Sensing – Stitching Process

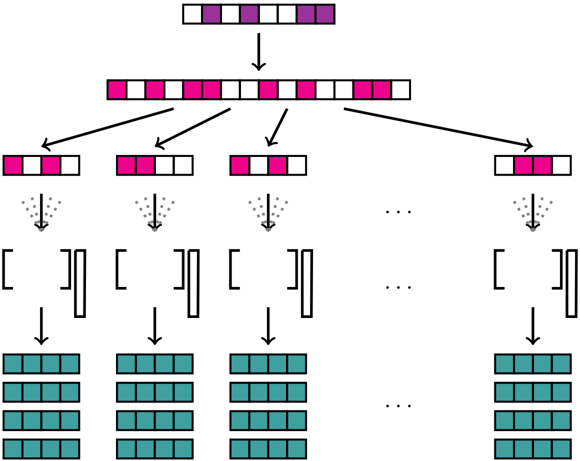


Tree decoding principles

- Every parity is linear combination of bits in preceding blocks
- Late parity bits offer better performance
- Early parity bits decrease decoding complexity
- Correct fragment is on list

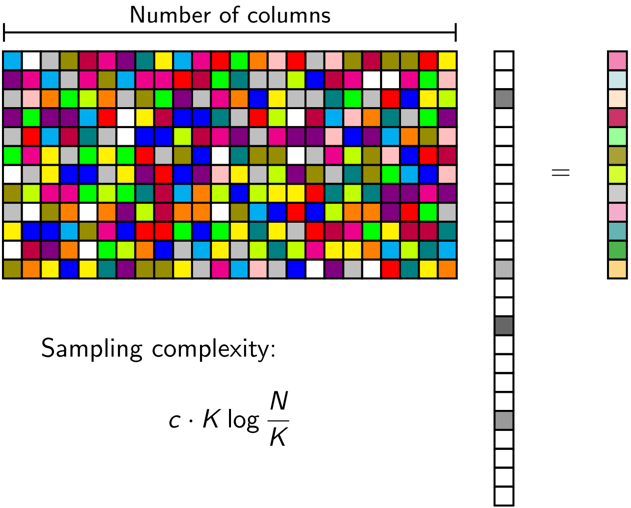


Coded Compressive Sensing – Divide and Conquer



- Data fragmentation and indexing
- Outer encoding for disambiguation

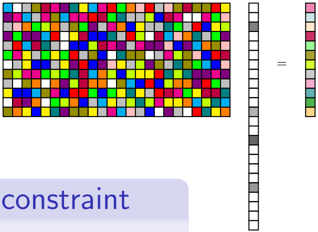
Vignette – Compressed Sensing



Sampling complexity:

$$c \cdot K \log \frac{N}{K}$$

CS Vignette – Basis Pursuit – LASSO



Optimization objective with sparsity constraint

When $\mathbf{\Phi}$ satisfies certain conditions, e.g., RIP, we can get a good estimate for sparse $\mathbf{s}_0$ by solving convex program

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \|\mathbf{y} - \mathbf{\Phi}\mathbf{s}\|_2 + \lambda\|\mathbf{s}\|_1$$

- Extensive analysis and wide applications
- LP, QP, ISTA w/o momentum, NNLS, etc.

CS Vignette – Underdetermined Linear Systems

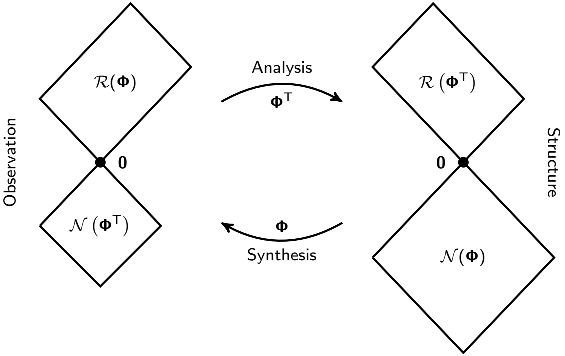
Optimization task

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{s}\|_2 \\ \text{subject to} \quad & \mathbf{\Phi s} = \mathbf{y} \end{aligned}$$

Composite iterative solution

$$\mathbf{z}^{(t)} = \mathbf{y} - \mathbf{\Phi s}^{(t)}$$

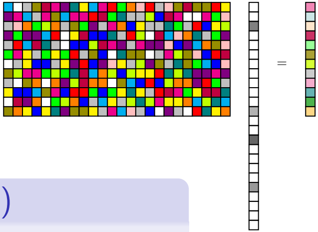
$$\mathbf{s}^{(t+1)} = \mathbf{\Phi}^{\mathsf{T}}\mathbf{z}^{(t)} + \mathbf{s}^{(t)}$$



Neumann series

$\mathbf{s}^{(t+1)} \to \mathbf{\Phi}^{\mathsf{T}}\left(\mathbf{\Phi\Phi}^{\mathsf{T}}\right)^{-1}\mathbf{y}$

CS Vignette – Iterative Soft Thresholding



Approximate message passing (AMP)

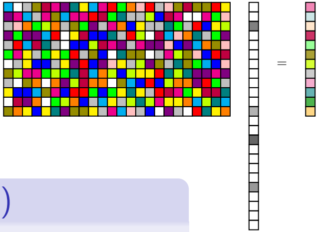
$$\mathbf{z}^{(t)} = \mathbf{y} - \mathbf{\Phi}\mathbf{s}^{(t)} + \overbrace{\frac{\mathbf{z}^{(t-1)}}{n}\|\mathbf{s}^{(t)}\|_0}^{\text{Onsager}}$$

$$\mathbf{s}^{(t+1)} = \boldsymbol{\eta}\left(\mathbf{\Phi}^{\mathsf{T}}\mathbf{z}^{(t)} + \mathbf{s}^{(t)}\right)$$

where $\boldsymbol{\eta}(\mathbf{s})_k = (|\mathbf{s}_k| - \alpha\lambda)_+ \operatorname{sgn}(\mathbf{s}_k)$, $\mathbf{s}^{(0)} = \mathbf{0}$, $\mathbf{z}^{(0)} = \mathbf{y}$

- Application to high-dimensional spaces
- Low complexity, scalable framework

CS Vignette – AMP



Approximate message passing (AMP)

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$$\mathbf{s}^{(t+1)} = \boldsymbol{\eta}\left(\boldsymbol{\Phi}^{\mathsf{T}}\mathbf{z}^{(t)} + \mathbf{s}^{(t)}\right)$$

where $\boldsymbol{\eta}(\mathbf{s})_k = (|\mathbf{s}_k| - \alpha\lambda)_+ \operatorname{sgn}(\mathbf{s}_k)$, $\mathbf{s}^{(0)} = \mathbf{0}$, $\mathbf{z}^{(0)} = \mathbf{y}$

- Application to high-dimensional spaces
- Low complexity, scalable framework

CCS – Approximate Message Passing

SPARCs for Unsourced Random Access

Alexander Fengler, Peter Jung, Giuseppe Caire

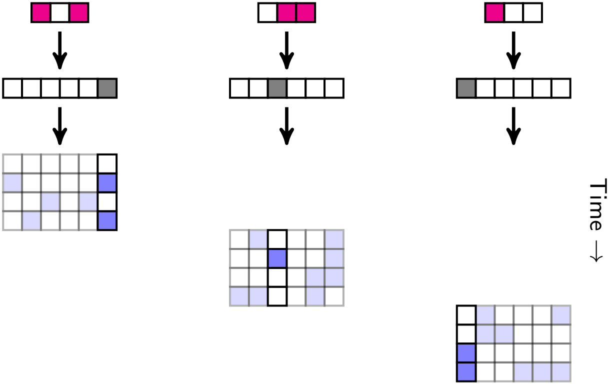
(Submitted on 18 Jan 2019)

This paper studies the optimal achievable performance of compressed sensing based unsourced random-access communication over the real AWGN channel. "Unsourced" means, that every user employs the same codebook. This paradigm, recently introduced by Polyanskiy, is a natural consequence of a very large number of potential users of which only a finite number is active in each time slot. The idea behind compressed sensing based schemes is that each user encodes his message into a sparse binary vector and compresses it into a real or complex valued vector using a random linear mapping. When each user employs the same matrix this creates an effective binary inner multiple-access channel. To reduce the complexity to an acceptable level the messages have to be split into blocks. An outer code is used to assign the symbols to individual messages. This division into sparse blocks is analogous to the construction of sparse regression codes (SPARCs), a novel type of channel codes, and we can use concepts from SPARCs to design efficient random-access codes. We analyze the asymptotically optimal performance of the inner code using the recently rigorized replica symmetric formula for the free energy which is achievable with the approximate message passing (AMP) decoder with spatial coupling. An upper bound on the achievable rates of the outer code is derived by classical Shannon theory. Together this establishes a framework to analyse the trade-off between SNR, complexity and achievable rates in the asymptotic infinite blocklength limit. Finite blocklength simulations show that the combination of AMP decoding, with suitable approximations, together with an outer code recently proposed by Amalladinne et. al. outperforms state of the art methods in terms of required energy-per-bit at lower decoding complexity.

Comments: 16 pages, 7 Figures
Subjects: **Information Theory (cs.IT)**
Cite as: **arXiv:1901.06234 [cs.IT]**
(or **arXiv:1901.06234v1 [cs.IT]** for this version)

- Connection between CCS indexing and sparse regression codes
- Circumvent slotting under CCS and dispersion effects
- Introduce denoiser tailored to CCS

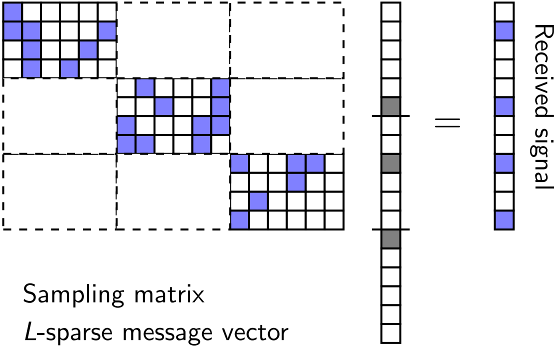
CCS Revisited



Columns are possible signals

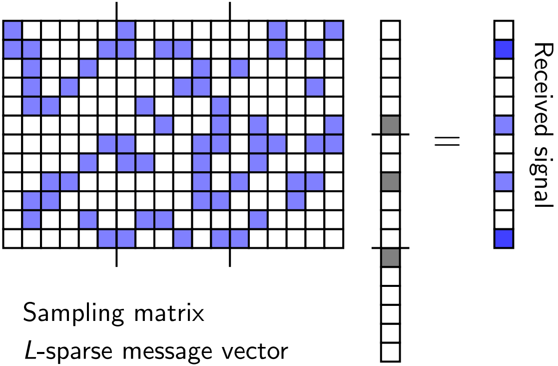
- Bit sequence split into L fragments
- Each bit + parity block converted to index in $\left[0, 2^{B/L} - 1\right]$
- Stack sub-codewords into $(n/L) \times 2^{B/L}$ sensing matrices

Coded Compressed Sensing – Unified View



- Slots produce block diagonal (unified) matrix
- Message is one-sparse per section
- Width of $\mathbf{A}$ is smaller: $L2^{B/L}$ instead of 2^B

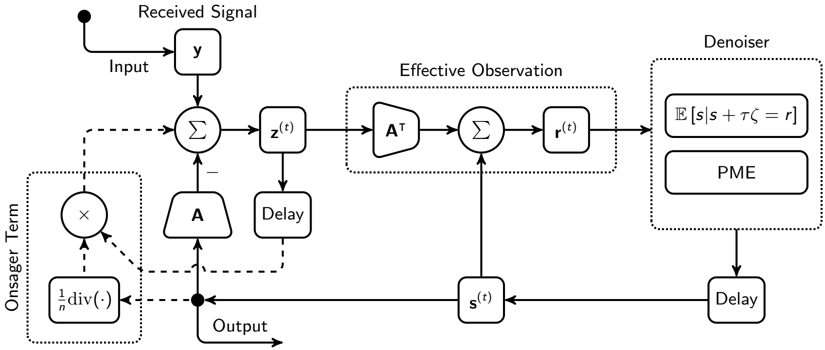
CCS – Full Sensing Matrix



- Complexity reduction due to narrower **A**
- Use full sensing matrix **A**
- Decode inner code with low-complexity AMP

A. Fengler, P. Jung and G. Caire. *SPARCs for unsourced Random access*. IEEE Trans. Information Theory, 2021.

CCS – AMP Architecture



CCS – Approximate Message Passing

Governing equations

- AMP algorithm iterates through

$$\mathbf{z}^{(t)} = \mathbf{y} - \mathbf{A}\mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right) + \underbrace{\frac{\mathbf{z}^{(t-1)}}{n} \operatorname{div} \mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Onsager correction}}$$

$$\mathbf{r}^{(t+1)} = \mathbf{A}^{\mathsf{T}}\mathbf{z}^{(t)} + \mathbf{D}\,\underbrace{\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Denoiser}}$$

Initial conditions $\mathbf{z}^{(0)} = \mathbf{0}$ and $\eta_0\left(\mathbf{r}^{(0)}\right) = \mathbf{0}$

- Application falls within framework for non-separable functions

Task

- Define denoiser and compute Onsager correction term

Marginal Posterior Mean Estimate (PME)

Proposed denoiser (Fengler, Jung, and Caire)

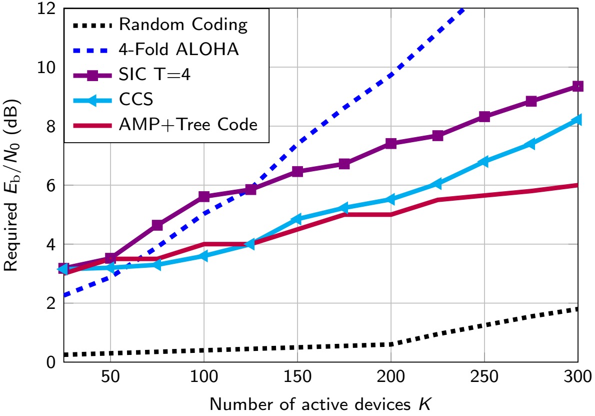
- State estimate based on Gaussian model

$$\hat{s}^{\mathrm{OR}}(q, r, \tau) = \mathbb{E}\left[s \middle| \sqrt{P_\ell} s + \tau \zeta = r \right]$$

$$= \frac{q \exp\left(- \frac{\left(r - \sqrt{P_\ell} \right)^2}{2\tau^2} \right)}{(1-q) \exp\left(- \frac{r^2}{2\tau^2} \right) + q \exp\left(- \frac{\left(r - \sqrt{P_\ell} \right)^2}{2\tau^2} \right)}$$

  with (essentially) uninformative prior $q = 1 - \left(1 - \frac{1}{m}\right)^K$ fixed
- $\boldsymbol{\eta}_t\left(\mathbf{r}^{(t)}\right)$ is aggregate of PME values
- τ_t is obtained from state evolution or $\tau_t^2 = \|\mathbf{z}^{(t)}\|^2 / n$

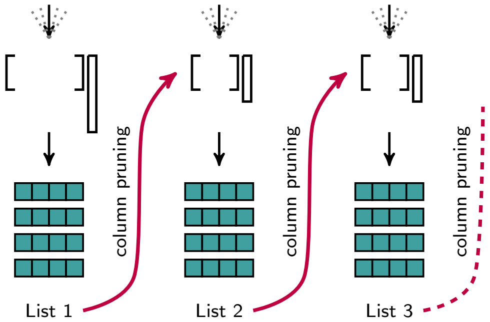
Performance of CCS Schemes



- V. K. Amalladinne, JFC and K. R. Narayanan. *A coded compressed sensing scheme for unsourced multiple access*. IEEE Trans. Information Theory, 2020.
- A. Fengler, P. Jung and G. Caire. SPARCs for unsourced random access. *IEEE Trans. Information Theory*, 2021.

Incorporating Lessons from Enhanced CCS

- Integrate outer code structure into inner decoding

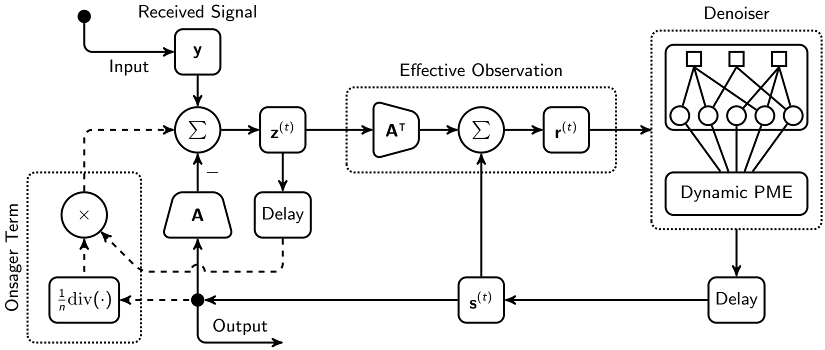


Challenges

- CCS-AMP inner decoding is not a sequence of hard decisions
- List size for CCS-AMP is effective length of index vector

V. K. Amalladinne, A. K. Pradhan, C. Rush, JFC, K. R. Narayanan. *Unsourced random access with coded compressed sensing: Integrating AMP and belief propagation*. IEEE Trans. Information Theory, 2022.

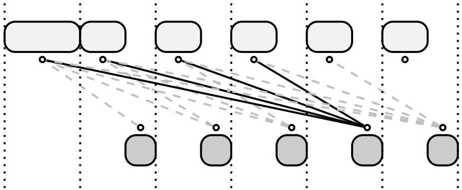
CCS – AMP Architecture with Outer Code



Redesigning Outer Code

Properties of original outer code

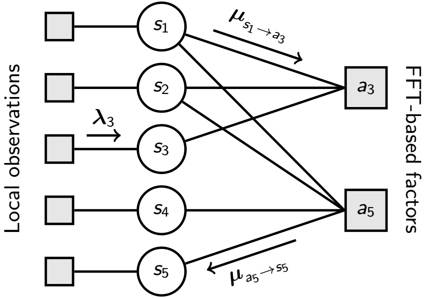
- Aimed at stitching message fragments together
- Works on short lists of K fragments
- Parities allocated to control growth and complexity



Challenges to integrate into AMP

1. Must compute beliefs for all possible fragments
2. Must provide pertinent information to inner AMP decoder
3. Should maintain ability to stitch outer code

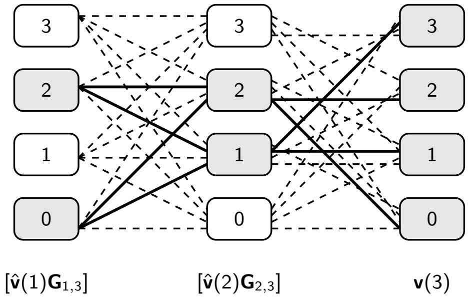
Factor Graph Interpretation of Outer Code



- Outer code with circular convolution structure

$$\boldsymbol{\mu}_{a_p \to s_\ell}\left([\hat{\mathbf{v}}(\ell)]_2\right) \propto \frac{1}{\left\|\mathbf{g}_{\ell,p}^{(g)}\right\|_0} \left(\mathsf{FFT}^{-1}\left(\prod_{s_j \in N(a_p) \setminus s_\ell} \mathsf{FFT}\left(\boldsymbol{\lambda}_{j,p}\right)\right)\right)(g)$$

Outer Code and Mixing

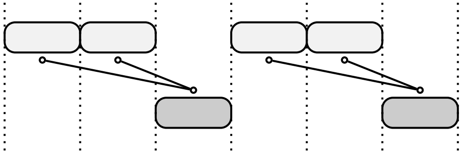


- Multiple devices on same graph
- Parity factor mix concentrated values
- Suggests triadic outer structure

Redesigning Outer Code

Solutions to integrate into AMP

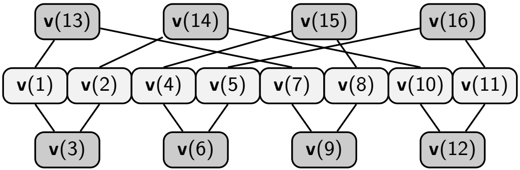
- Parity bits are generated over Abelian group amenable to FWHT or FFT
- Discrimination power proportional to # parities



New design strategy

1. Information sections with parity bits interspersed in-between
2. Parity over two blocks (triadic dependencies)

Belief Propagation – Message Passing Rules



- Message from check node a_p to variable node $s \in N(a_p)$:

$$\boldsymbol{\mu}_{a_p \to s}(k) = \sum_{\mathbf{k}_{a_p} : k_p = k} \mathcal{G}_{a_p}\left(\mathbf{k}_{a_p}\right) \prod_{s_j \in N(a_p) \setminus s} \boldsymbol{\mu}_{s_j \to a_p}(k_j)$$

- Message from variable node s_ℓ to check node $a \in N(s)$:

$$\boldsymbol{\mu}_{s_\ell \to a}(k) \propto \boldsymbol{\lambda}_\ell(k) \prod_{a_p \in N(s_\ell) \setminus a} \boldsymbol{\mu}_{a_p \to s_\ell}(k)$$

- Estimated marginal distribution

$$p_{s_\ell}(k) \propto \boldsymbol{\lambda}_\ell(k) \prod_{a \in N(s_\ell)} \boldsymbol{\mu}_{a \to s_\ell}(k)$$

Approximate Message Passing Algorithm

Updated equations

AMP two-step algorithm

$$\mathbf{z}^{(t)} = \mathbf{y} - \mathbf{A}\mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right) + \underbrace{\frac{\mathbf{z}^{(t-1)}}{n} \operatorname{div} \mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Correction}}$$

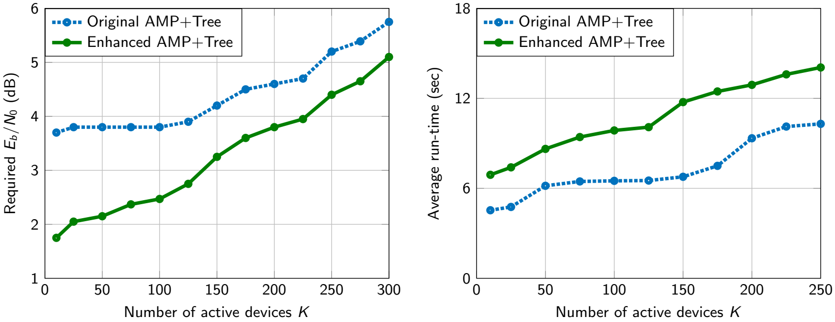
$$\mathbf{r}^{(t+1)} = \mathbf{A}^{\mathsf{T}}\mathbf{z}^{(t)} + \mathbf{D}\,\underbrace{\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Denoiser}}$$

Initial conditions $\mathbf{z}^{(0)} = \mathbf{0}$ and $\eta_0\left(\mathbf{r}^{(0)}\right) = \mathbf{0}$

- Denoiser is BP estimate from factor graph
- Message passing uses fresh effective observation $\mathbf{r}$
- Fewer rounds than shortest cycle on factor graph
- Close to PME, but incorporating beliefs from outer code

R. Berthier, A. Montanari, and P.-M. Nguyen. *State evolution for approximate message passing with non-separable functions*. Information and Inference, 2020.

Preliminary Performance Enhanced CCS

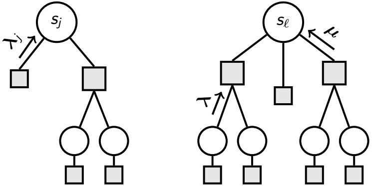


- Performance improves significantly with enhanced CCS-AMP decoding
- Computational complexity is approximately maintained
- Reparametrization may offer additional gains in performance?

CCS and AMP Summary

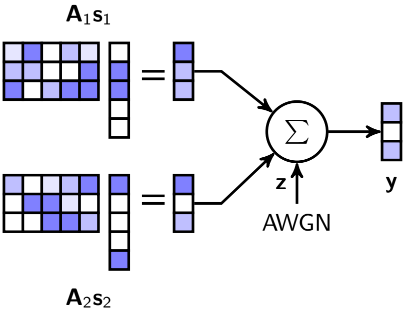
Summary

- New connection between CCS and AMP
- Natural application of BP on factor graph as denoiser
- Outer code design depends on sparsity
    1. Degree distributions (small graph)
    2. Message size (birthday problem)
    3. Final step is disambiguation
- Many theoretical and practical challenges/opportunities exist



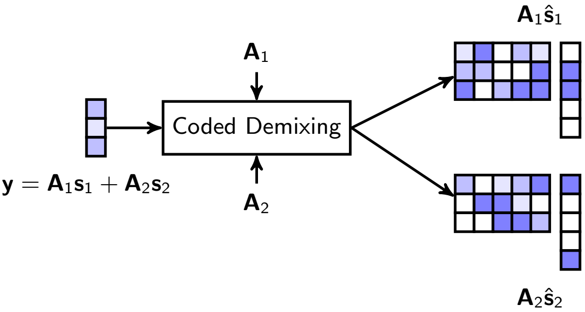
Coding plays increasingly central role in large-scale CS

CCS via Coded Demixing



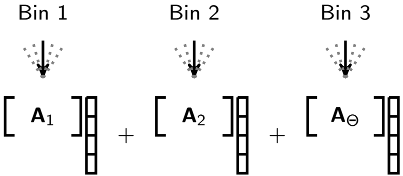
- CCS can be extended to accommodate multiple classes of heterogeneous users
- Each class of users employs its own sensing matrix and factor graph for message encoding
- Every class transmits its signal concurrently over GMAC

CCS via Coded Demixing

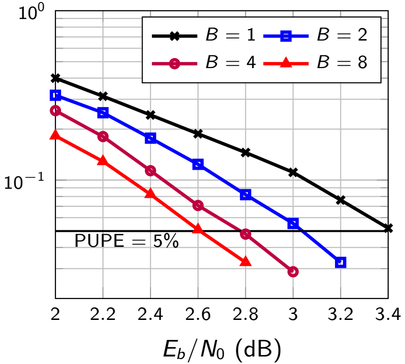


- Through coded demixing, signals from various classes may be separated by receiver and decoded individually
- Efficient AMP-based algorithm to recover signals from different classes
- Requires signals to be sparse and the sensing matrices to have low cross-coherence

Coded Demixing for Single-Class URA

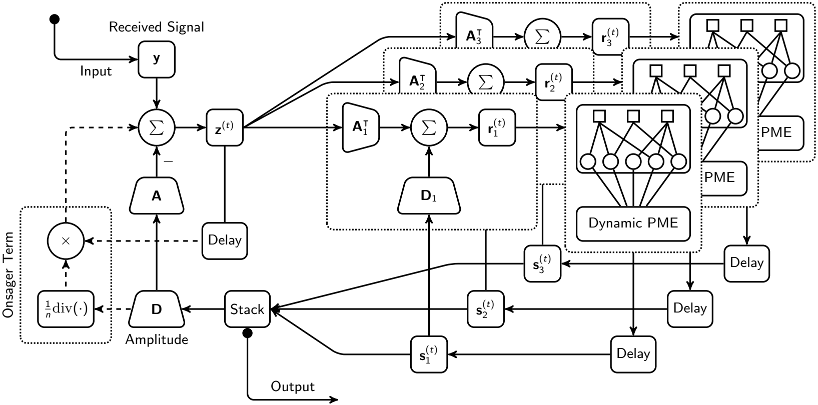


- Create multiple bins with (incoherent) matrices
- Devices pick a bucket randomly and use CCS-AMP encoding
- Perform joint demixing CCS-AMP decoding at access point

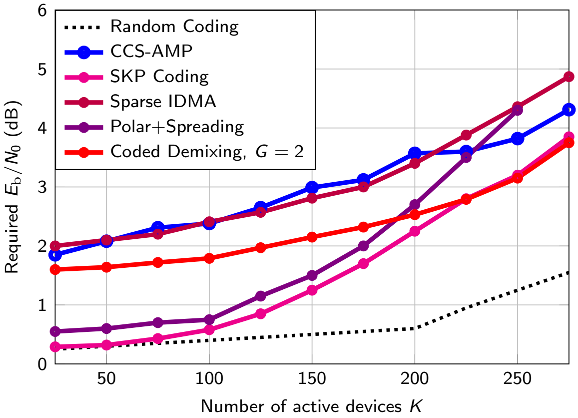


J. R. Ebert, V. K. Amalladinne, S. Rini, JFC, K. R. Narayanan. *Coded demixing for unsourced random access*. IEEE Trans. Signal Processing, 2022.

CCS – Coded Demixing Architecture



Performance of CCS-AMP versus Previous Schemes



- Sparse interleave division multiple access (IDMA) by A. K. Pradhan, V. Amalladinne, A. Vem, K. R. Narayanan and JFC
- Sparse Kronecker-product (SKP) coding by Z. Han, X. Yuan, C. Xu, S. Jiang and X. Wang

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- M. Bayati and A. Montanari. *The dynamics of message passing on dense graphs, with applications to compressed sensing*. IEEE Trans. Information Theory, 2011.
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- C. Rush, A. Greig, and R. Venkataramanan. Capacity-achieving sparse superposition codes via approximate message passing decoding. *IEEE Trans. Information Theory*, 2017.
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- A. Fengler, P. Jung and G. Caire. SPARCs for unsourced random access. *IEEE Trans. Information Theory*, 2021.
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- J. R. Ebert, V. K. Amalladinne, S. Rini, J.-F. Chamberland, K. R. Narayanan. Coded demixing for unsourced random access. *IEEE Transactions on Signal Processing*, 2022.
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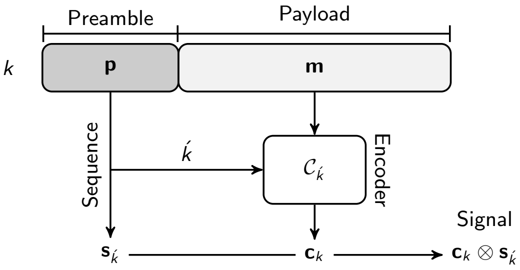
Part III

Sparsifying Collisions: Spread Unsourced Random Access with Tensor/Hadamard Constructions

Spreading or Stochastic Binning with Shadowing



- Message is partitioned into two parts $(\mathbf{p}, \mathbf{m})$
- Preamble $\mathbf{p}$ selects spreading sequence
- Payload $\mathbf{m}$ is encoded using traditional code: Polar/LDPC
- Tensor/Hadamard product is performed to create signal
  Note: $\mathbf{c}_k \otimes \mathbf{s}_{\acute{k}}$ or $(\mathbf{c}_k \otimes \mathbf{1}) \circ \mathbf{s}_{\acute{k}}$

Signal Structure and Energy Detector

- Outer product representation of sent signal

$$\mathbf{X} = \mathbf{s} \cdot \mathbf{c}^{\mathsf{T}} = \begin{bmatrix} | & | & & | \\ \mathbf{s}\, c_1 & \mathbf{s}\, c_2 & \cdots & \mathbf{s}\, c_m \\ | & | & & | \end{bmatrix} \qquad \mathbf{s}^H\, \mathbf{X} = \|\mathbf{s}\|_2^2\, \mathbf{c}^{\mathsf{T}}$$

- Outer product representation of received signal

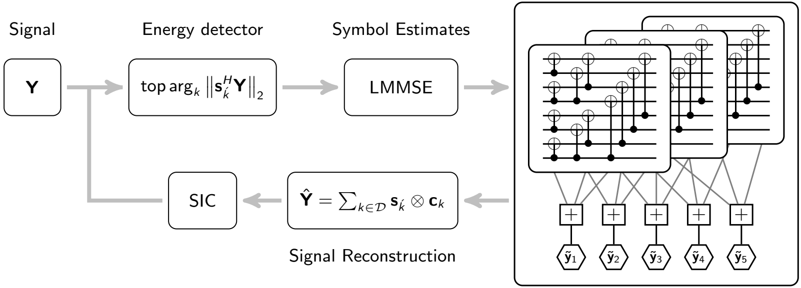
$$\mathbf{Y} = \sum_k \mathbf{X}_k + \mathbf{Z} = \sum_k \mathbf{s}_{\acute{k}} \cdot \mathbf{c}_k^{\mathsf{T}} + \mathbf{Z}$$

$$\mathbf{s}_{\acute{k}}^H \mathbf{Y} = \|\mathbf{s}_k\|_2^2\, \mathbf{c}_k^{\mathsf{T}} + \underbrace{\sum_{\ell \neq k} \left\langle \mathbf{s}_{\acute{k}}, \mathbf{s}_{\acute{\ell}} \right\rangle \mathbf{c}_\ell^{\mathsf{T}}}_{\text{Noise like}} + \mathbf{s}_{\acute{k}}^H \mathbf{Z}$$

- Energy detector for sequence identification

$$\text{Set of active sequence} = \underset{k}{\text{top arg}} \left\| \mathbf{s}_{\acute{k}}^H \mathbf{Y} \right\|_2$$

Joint Decoding Architecture



1. Sequence identification
2. Symbol estimation
3. Bank of single-user decoders
4. Signal reconstruction
5. Successive interference cancellation

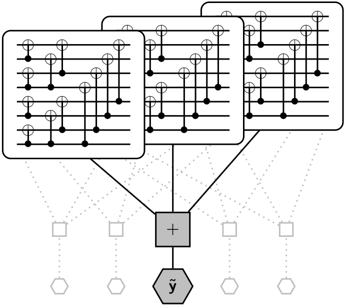
Intuition Behind Symbol Estimation

Measurement structure
(w/o collisions)

$$\mathbf{Y} = \begin{bmatrix} | & | & & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_m \\ | & | & & | \end{bmatrix}$$

$$= \sum_k \begin{bmatrix} | \\ \mathbf{s}_{\acute{k}} \\ | \end{bmatrix} \begin{bmatrix} - \mathbf{c}_k - \end{bmatrix} + \mathbf{Z}$$

$$= \mathbf{SC} + \mathbf{Z}$$



Approximate structure

$$\mathbf{Y} \approx \mathbf{S}_{\mathcal{D}}\mathbf{C} + \mathbf{Z}$$

$$\approx \sum_{k \in \mathcal{D}} \begin{bmatrix} | \\ \mathbf{s}_{\acute{k}} \\ | \end{bmatrix} \begin{bmatrix} - \mathbf{c}_k - \end{bmatrix} + \mathbf{Z}$$

- Covariance

$$\left(\mathbf{S}_{\mathcal{D}}\mathbf{S}_{\mathcal{D}}^H + \mathbf{I}\right)$$

- LMMSE

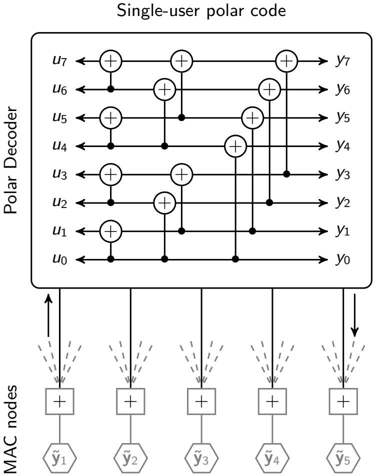
$$\hat{\mathbf{C}} \approx \mathbf{S}_{\mathcal{D}}^H \left(\mathbf{S}_{\mathcal{D}}\mathbf{S}_{\mathcal{D}}^H + \mathbf{I}\right)^{-1} \mathbf{Y}$$

Spreading or Stochastic Binning with Shadowing

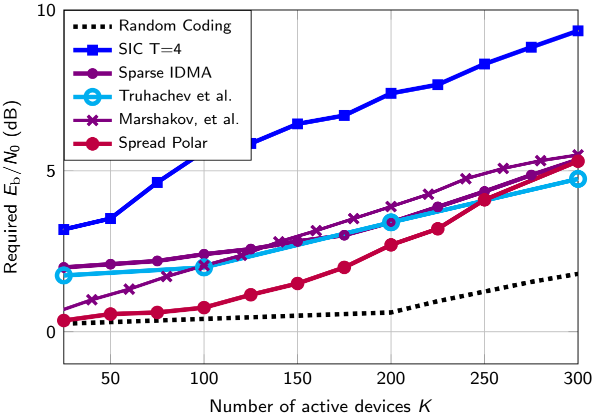
- Polar code
- Single-user likelihoods based on estimated rows of

$$\hat{\mathbf{C}} \approx \mathbf{S}_{\mathcal{D}}^{H} \left(\mathbf{S}_{\mathcal{D}} \mathbf{S}_{\mathcal{D}}^{H} + \mathbf{I} \right)^{-1} \mathbf{Y}$$

- Joint successive cancellation within decoding loop
- CRC added to codewords
- Sequences dictate frozen bits
- LMMSE can be tuned to account for collisions
- Framework can accommodate soft LDPC symbol estimates



Spread URA Single-Antenna



- *Spread polar* outperforms Irregular Repetition Slotted ALOHA (IRSA) polar by E. Marshakov, G. Balitskiy, K. Andreev, A. Frolov
- Low complexity scheme by D. Truhachev, M. Bashir, A. Karami, E. Nassaji performs well for large population

Signal Structure Revisited

- Note: $\mathbf{c} \otimes \mathbf{s}$ or $(\mathbf{c} \otimes \mathbf{1}) \circ \mathbf{s}^{+}$
- New representation of sent signal

$$\mathbf{X} = (\mathbf{c} \otimes \mathbf{1}) \circ \mathbf{s}^{+} \Rightarrow \begin{bmatrix} | & | & & | \\ \mathbf{s}^{+}(1)\, c_1 & \mathbf{s}^{+}(2)\, c_2 & \cdots & \mathbf{s}^{+}(m)\, c_m \\ | & | & & | \end{bmatrix}$$

  with a different spreading column for every coded symbol
- One LMMSE matrix inversion per coded symbol period j

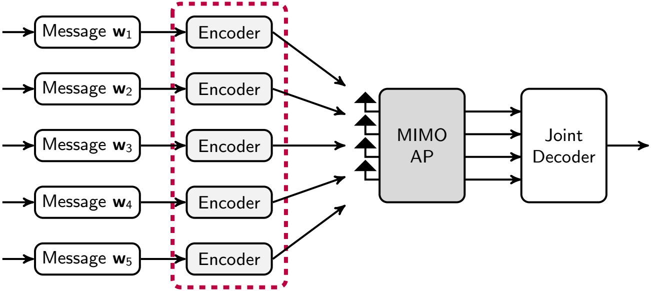
$$\hat{\mathbf{c}}(j) \approx \mathbf{S}_{j,\mathcal{D}}^{H} \left(\mathbf{S}_{j,\mathcal{D}} \mathbf{S}_{j,\mathcal{D}}^{H} + \mathbf{I} \right)^{-1} \mathbf{y}_j$$

- Framework becomes more general and subsumes IRSA
- Match filter versus LMMSE
  - Buying performance at expense of complexity
  - Model becomes more brittle to fine synchronization
- Random subset of sequence set precludes CDMA-style designs

Part IV

Quasi-Static Massive MIMO Unsourced Random Access Channels

Massive MIMO URA – Quasi-Static Channel



Quasi-static signal model

- Signal received at time instant t within slot ℓ

$$\mathbf{y}(t,\ell) = \sum_{k=1}^{K} \mathbf{x}_k(t,\ell)\mathbf{h}_k + \mathbf{z}(t,\ell)$$

- Number of receive antennas $M \gg 1$
- Fading does NOT change within URA frame

Massive MIMO URA – Quasi-Static Channel

Problem formulation

- Noisy MMV support recovery

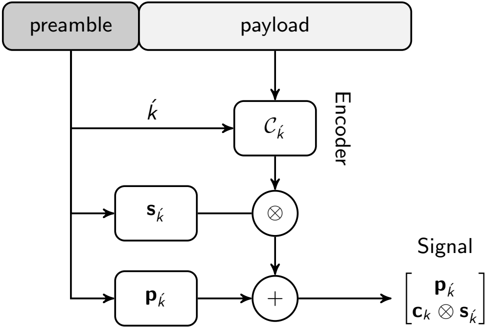
$$\mathbf{Y} = \sum_{k} \mathbf{x}_k \cdot \mathbf{h}_k^{\mathsf{T}} + \mathbf{Z}$$

- Channel coefficients are not known
- Number of antennas M is large
- Channel is quasi-static

Possible URA design strategies

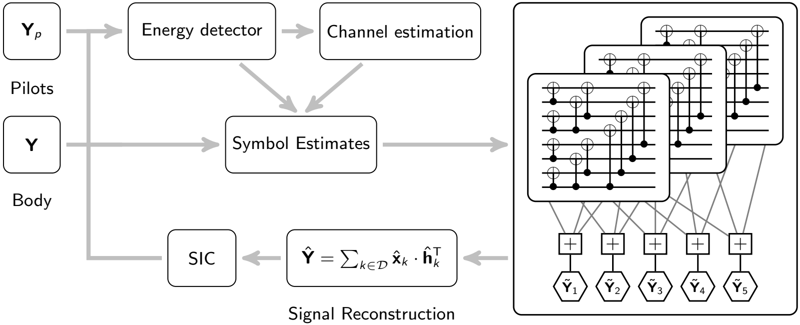
- Collisions may not be as much of an issue
- Complexity must be managed
- Strategies with pilots seem advantageous

Proposed Encoding – Pilot plus Spreading



- Encoding similar to spread URA, albeit with pilots
- Pilot sequence used for activity detection and channel estimation
- Payload $\mathbf{m}$ is encoded using traditional code: Polar/LDPC

Joint Decoding Architecture MIMO



Energy Detector

$$\text{Pilot set} = \underset{k}{\text{top arg}} \left\| \mathbf{p}_{\acute{k}}^H \mathbf{Y}_p \right\|_2$$

LMMSE channel estimation

$$\hat{\mathbf{H}} = \left(\sigma^2 \mathbf{I} + \hat{\mathbf{P}}^H \hat{\mathbf{P}} \right)^{-1} \hat{\mathbf{P}}^H \mathbf{Y}_p$$

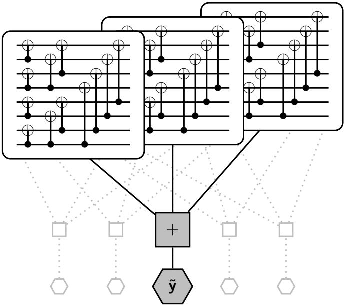
Intuition Behind MIMO Symbol Estimation

Measurement structure

For symbol period j

$$\begin{aligned}
&\mathbf{Y}[\mathbf{n}_j, m] \\
&\approx \mathbf{S}_j \operatorname{diag}(\mathbf{c}_j)\hat{\mathbf{H}}[:, m] + \mathbf{Z}[\mathbf{n}_j, m] \\
&= \mathbf{S}_j \operatorname{diag}\left(\hat{\mathbf{H}}[:, m]\right) \mathbf{c}_j + \mathbf{Z}[\mathbf{n}_j, m]
\end{aligned}$$

where $\mathbf{n}_j = [(j-1)L + 1 \colon jL]$ spread slice and m is antenna index



Stacked vector and LMMSE estimates

$$\begin{bmatrix} \mathbf{Y}[\mathbf{n}_j, 1] \\ \vdots \\ \mathbf{Y}[\mathbf{n}_j, M] \end{bmatrix} = \begin{bmatrix} \mathbf{S}_j \operatorname{diag}\left(\hat{\mathbf{H}}[:, 1]\right) \\ \vdots \\ \mathbf{S}_j \operatorname{diag}\left(\hat{\mathbf{H}}[:, M]\right) \end{bmatrix} \mathbf{c}_j + \begin{bmatrix} \mathbf{Z}[\mathbf{n}_j, 1] \\ \vdots \\ \mathbf{Z}[\mathbf{n}_j, M] \end{bmatrix}$$

Alternate Scheme – Tensor-Based Modulation

Code Construction

Codebook is created based on tensors

$$\{\mathbf{x}_1 \otimes \mathbf{x}_2 \otimes \cdots \otimes \mathbf{x}_d : \mathbf{x}_1 \in \mathcal{C}_1, \mathbf{x}_2 \in\in \mathcal{C}_2, \ldots \mathbf{x}_d \in \mathcal{C}_d\}$$

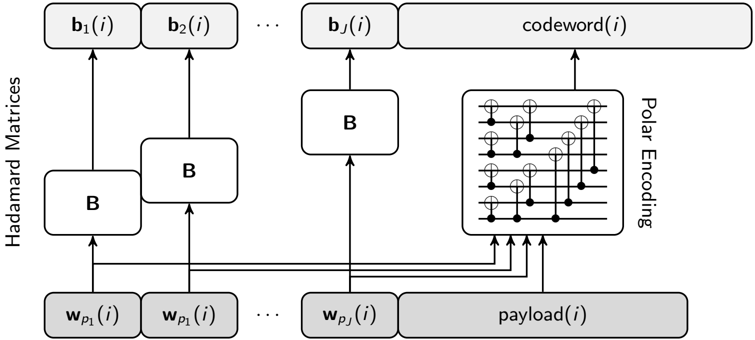
Received signal is sum of K rank-1 tensors plus noise

$$\sum_k \mathbf{x}_{1,k} \otimes \mathbf{x}_{2,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \otimes \mathbf{h}_k + \mathbf{z}$$

- Decode with canonical polyadic decomposition (CPD)
- Iterative nonlinear least square algorithm on flattened outer products
- Pilots are not used in this scheme

A. Decurninge, I. Land, and M. Guillaud. *Tensor-based modulation for unsourced massive random access*. IEEE Wireless Communications Letters, 2021.

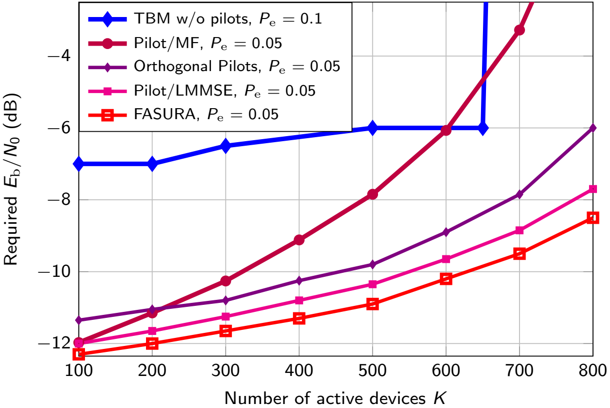
Alternate Scheme – Orthogonal Pilots



- Hadamard pilots for fast processing: detection and estimation
- Polar code plus cyclic redundancy check (CRC)
- Excellent performance versus complexity tradeoff

M. J. Ahmadi and T. M. Duman. *Unsourced random access with a massive MIMO receiver using multiple stages of orthogonal pilots*. ISIT, 2022.

Spread URA Single-Antenna



- Tensor-Based Modulation by A. Decurninge, I. Land, & M. Guillaud
- Orthogonal Pilots by M. J. Ahmadi & T. M. Duman
- Pilot/MF by A. Fengler, O. Musa, P. Jung, & G. Caire
- FASURA by M. Gkagkos, K. R. Narayanan, J.-F. Chamberland, and C. N. Georghiades

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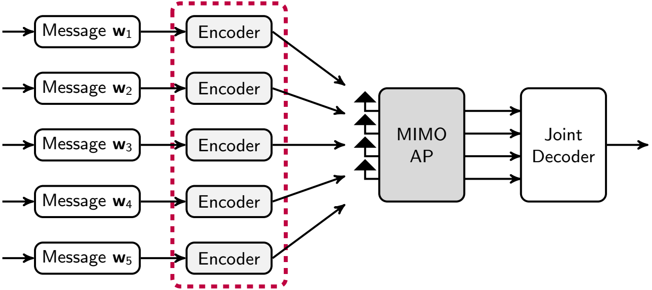
Part V

Fast Fading Massive MIMO Unsourced Random Access Channels

Massive MIMO-URA – Fast Fading Channel



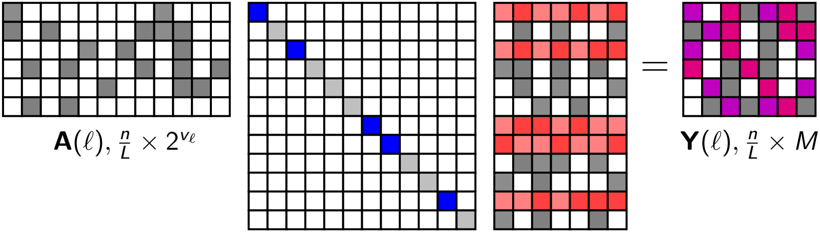
Signal model

- Signal received at time instant t with slot ℓ

$$\mathbf{y}(t, \ell) = \sum_{k=1}^{K} \mathbf{x}_k(t, \ell)\mathbf{h}_k(\ell) + \mathbf{z}(t, \ell)$$

- Number of receive antennas $M \gg 1$
- Block fading – channel does not change within CCS slot
- Spatial correlation negligible – $\mathbf{h}_k(\ell) \sim \mathcal{CN}(0, \mathbf{I}_M)$

Multiple Measurement Vector – CS Interpretation



$\mathbf{A}(\ell), \frac{n}{L} \times 2^{v_\ell}$

$\mathbf{\Gamma}(\ell) = \mathrm{diag}(\boldsymbol{\gamma}(\ell))$

$\boldsymbol{\gamma}(\ell) \in \{0, 1\}^{2^{v_\ell}}$

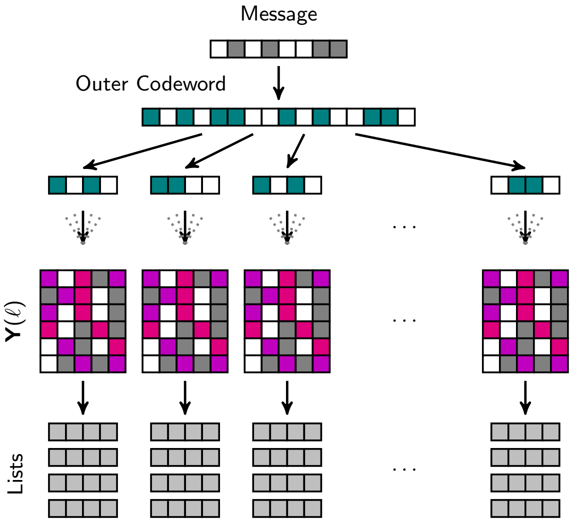
$\|\boldsymbol{\gamma}(\ell)\|_0 = K$

$\mathbf{H}(\ell), 2^{v_\ell} \times M$

$\mathbf{Y}(\ell), \frac{n}{L} \times M$

- Received signal during slot ℓ: $\mathbf{Y}(\ell) = \mathbf{A}(\ell)\mathbf{\Gamma}(\ell)\mathbf{H}(\ell) + \mathbf{Z}(\ell)$
- Column $\mathbf{y}_i(\ell)$ of $\mathbf{Y}(\ell)$ is the signal received at antenna i during slot ℓ
- $\mathbf{H}(\ell)$ has entries drawn i.i.d. from $\mathcal{CN}(0, 1)$

Divide-and-Conquer, again – Outer Tree Code



Covariance-Based Estimation

Idea

Since channel vectors are Gaussian, columns $\mathbf{y}_i(\ell)$ of $\mathbf{Y}(\ell)$ are i.i.d. Gaussian $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_\ell)$

Computing covariance matrix

$$\begin{aligned}
\mathbf{\Sigma}_\ell &= \mathbb{E}\left[\mathbf{y}_i(\ell)\mathbf{y}_i(\ell)^H\right] = \frac{1}{M}\mathbb{E}\left[\mathbf{Y}(\ell)\mathbf{Y}(\ell)^H\right] \\
&= \frac{1}{M}\mathbb{E}\left[(\mathbf{A}(\ell)\mathbf{\Gamma}(\ell)\mathbf{H}(\ell) + \mathbf{Z}(\ell))\ (\mathbf{A}(\ell)\mathbf{\Gamma}(\ell)\mathbf{H}(\ell) + \mathbf{Z}(\ell))^H\right] \\
&= \frac{1}{M}\left(\mathbf{A}(\ell)\mathbf{\Gamma}(\ell)\underbrace{\mathbb{E}\left[\mathbf{H}(\ell)\mathbf{H}(\ell)^H\right]}_{M\ \mathbf{I}_{n/L}}\mathbf{\Gamma}(\ell)^H\mathbf{A}(\ell)^H + \underbrace{\mathbb{E}\left[\mathbf{Z}(\ell)\mathbf{Z}(\ell)^H\right]}_{N_0 M\ \mathbf{I}_{n/L}}\right) \\
&= \mathbf{A}(\ell)\mathbf{\Gamma}(\ell)\mathbf{A}(\ell)^H + N_0\mathbf{I}_{n/L}
\end{aligned}$$

Covariance-Based Estimation

Measurement $\mathbf{Y}(\ell)$ is Gaussian with covariance

$$\boldsymbol{\Sigma}_\ell = \frac{1}{M}\mathbb{E}\left[\mathbf{Y}(\ell)\mathbf{Y}(\ell)^H\right] = \mathbf{A}(\ell)\boldsymbol{\Gamma}(\ell)\mathbf{A}(\ell)^H + N_0\mathbf{I}$$

Use empirical average

- Let $\hat{\boldsymbol{\Sigma}}_{\mathbf{Y}(\ell)} = \frac{1}{M}\mathbf{Y}(\ell)\mathbf{Y}(\ell)^H$ be empirical covariance matrix
- Constrained ML estimate of $\gamma(\ell)$ given by

$$\begin{aligned}
\gamma^*(\ell) &= \underset{\boldsymbol{\gamma}(\ell)\in\mathbb{R}_+^{2^{v_\ell}}}{\arg\max} \log p(\mathbf{Y}(\ell)|\gamma(\ell)) \\
&= \underset{\boldsymbol{\gamma}(\ell)\in\mathbb{R}_+^{2^{v_\ell}}}{\arg\max} \frac{1}{M}\sum_{i=1}^{M} \log p(\mathbf{y}_i(\ell)|\gamma(\ell)) \\
&= \underset{\boldsymbol{\gamma}(\ell)\in\mathbb{R}_+^{2^{v_\ell}}}{\arg\min} \left(\log|\boldsymbol{\Sigma}_\ell| + \mathrm{trace}\left(\boldsymbol{\Sigma}_\ell^{-1}\hat{\boldsymbol{\Sigma}}_{\mathbf{Y}(\ell)}\right)\right)
\end{aligned}$$

Iterative Updates Based on Sherman-Morrison Identity

Algorithm Covariance Based Estimation via Coordinate Descent

Inputs: Sample covariance $\hat{\mathbf{\Sigma}}_{\mathbf{Y}(\ell)} = \frac{1}{M}\mathbf{Y}(\ell)\mathbf{Y}(\ell)^H$
Initialize: $\mathbf{\Sigma}_\ell = N_0\mathbf{I}, \gamma(\ell) = 0$
for $i = 1, 2, \ldots$ **do**
  for $k = 1, 2, \cdots, 2^{v_\ell}$ **do**
    Set $d^* = \frac{\mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}(\hat{\mathbf{\Sigma}}_{\mathbf{Y}(\ell)}\mathbf{\Sigma}_\ell^{-1} - \mathbf{I}_n)\mathbf{a}_k(\ell)}{(\mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell))^2}$
    Update $\gamma_k(\ell) \leftarrow \max\{\gamma_k(\ell) + d^*, 0\}$
    Update $\mathbf{\Sigma}_\ell^{-1} \leftarrow \mathbf{\Sigma}_\ell^{-1} - \frac{d^* \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell)\mathbf{a}_k(\ell)^H\mathbf{\Sigma}_\ell^{-1}}{1 + d^* \mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell)}$

Output: Estimate $\gamma(\ell)$

- Component-wise maximization of the log-likelihood cost function
- Guaranteed to converge to at least a local minimum
- Good empirical performance

A Fengler, S Haghighatshoar, P Jung, G Caire. *Non-Bayesian activity detection, large-scale fading coefficient estimation, and unsourced random access with a massive MIMO receiver*. Trans. Information Theory, 2021

Sherman-Morrison plus Tree Pruning

Algorithm Covariance Based Estimation via Coordinate Descent & SCLD

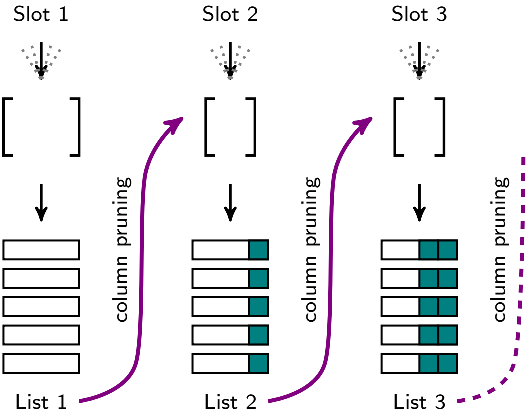
Inputs: Sample covariance $\hat{\mathbf{\Sigma}}_{\mathbf{Y}(\ell)} = \frac{1}{M}\mathbf{Y}(\ell)\mathbf{Y}(\ell)^H$
Initialize: $\mathbf{\Sigma}_\ell = N_0\mathbf{I}, \boldsymbol{\gamma}(\ell) = 0$
for $i = 1, 2, \ldots$ **do**
  for $k \in \mathcal{S}_\ell$ **do**
    Set $d^* = \frac{\mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}(\hat{\mathbf{\Sigma}}_{\mathbf{Y}(\ell)}\mathbf{\Sigma}_\ell^{-1} - \mathbf{I}_n)\mathbf{a}_k(\ell)}{(\mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell))^2}$
    Update $\gamma_k(\ell) \leftarrow \max\{\gamma_k(\ell) + d^*, 0\}$
    Update $\mathbf{\Sigma}_\ell^{-1} \leftarrow \mathbf{\Sigma}_\ell^{-1} - \frac{d^* \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell)\mathbf{a}_k(\ell)^H\mathbf{\Sigma}_\ell^{-1}}{1 + d^* \mathbf{a}_k(\ell)^H \mathbf{\Sigma}_\ell^{-1}\mathbf{a}_k(\ell)}$

Output: Estimate $\boldsymbol{\gamma}(\ell)$

- Descent performed only over subset $\mathcal{S}_\ell \subseteq [2^{v_\ell}]$ of columns in $\mathbf{A}(\ell)$
- $\mathcal{S}_\ell$ supplied by the outer tree decoder as side information
- Significant improvements in performance and complexity

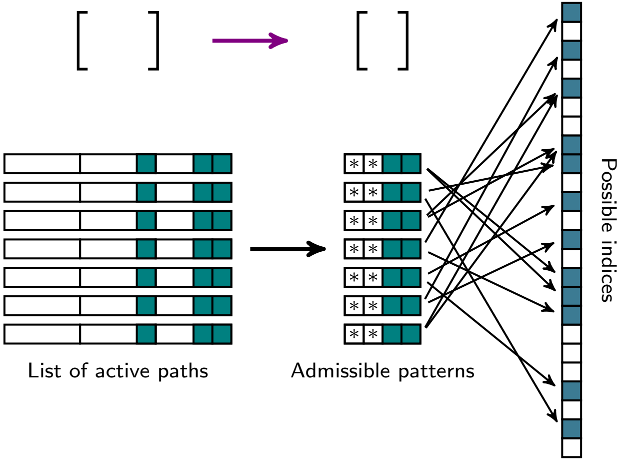
V. K. Amalladinne, J. R. Ebert, JFC, and K. R. Narayanan. *An enhanced decoding algorithm for coded compressed sensing with applications to unsourced random access*. MDPI Sensors, 2022.

Successive Cancellation List Decoding



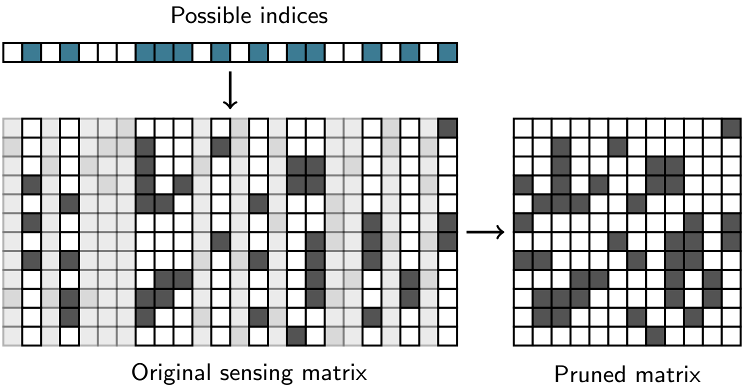
- Active partial paths determine possible parity patterns
- Admissible indices for next slot determined by possible parities
- Inadmissible columns can be pruned before CS algorithm

Outer Tree Decoding – Dimensionality Reduction



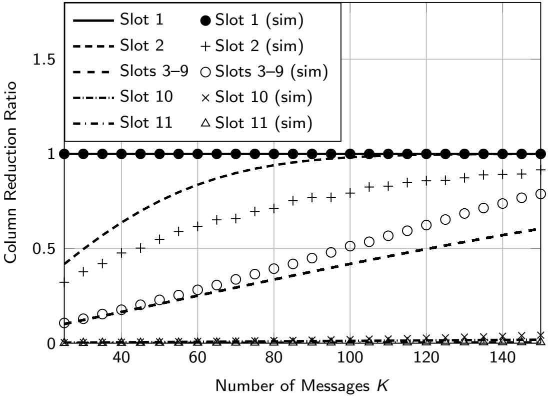
- Every surviving path produces parity pattern
- Only fragments with these pattern can appear in subsequent slot
- On average, there are $K(1 + \mathrm{E}[P_\ell])$ active paths

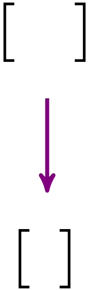
Outer Tree Decoding with Column Pruning



- For K small, width of sensing matrix is greatly reduced
- Actual sensing matrix is determined dynamically at run time
- Complexity of CS algorithm becomes much smaller

Expected Column Reduction Ratio



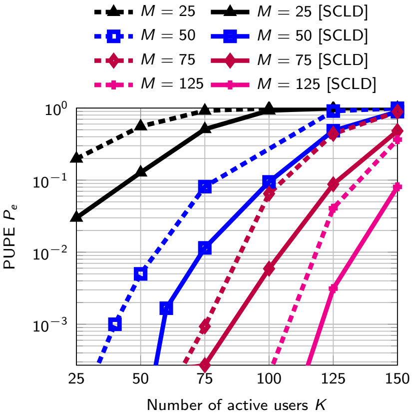


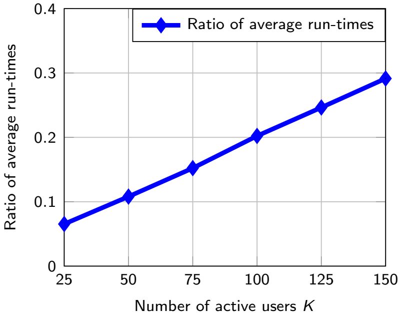
- Parity allocation parameters, with $w_\ell + p_\ell = 15$,

$$(p_1, p_2, \ldots, p_{10}) = (6, 8, 8, 8, 8, 8, 8, 8, 13, 15)$$

- Pruning is more pronounced at later stages
- Effective width of sensing matrix is greatly reduced

Performance Comparison





- Number of antennas reduced by 23% when $K = 100$
- Gains in computational complexity more pronounced when K modest
- Reparametrization may offer additional gains

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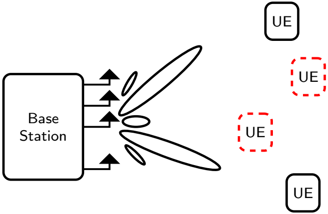
Part VI

Providing Feedback: Acknowledgments within URA

Feedback in Unsourced Random Access

- Existing URA schemes do not acknowledge reception of messages
- Acknowledgement (ACK) from BS is critical in certain applications
- Challenges of feedback in URA:
  - BS does not know which user sent each message
  - Large number of users

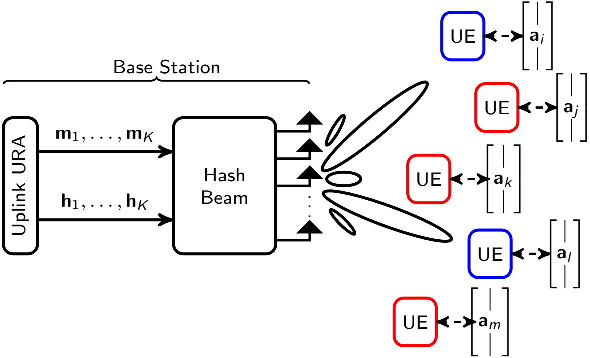


Section goal

Develop mechanism to acknowledge successful messages within URA

J. R. Ebert, K. R. Narayanan, and JFC. *HashBeam: Enabling feedback through downlink beamforming in unsourced random access*. arXiv, 2022.

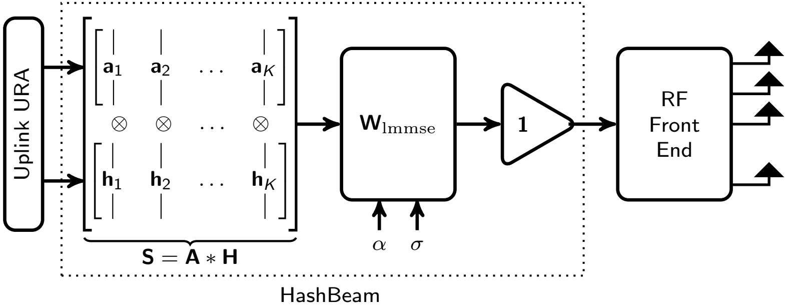
HashBeam: Feedback via Downlink Beamforming



Downlink beamforming in URA

- BS has channel estimate $\mathbf{h}_i$ and message $\mathbf{m}_i$ for every decoded UE
- UE i and BS compute hash $\mathbf{a}_i = f(\mathbf{m}_i)$, $f : \{0,1\}^B \rightarrow \mathbb{C}^L$
- $\mathbf{s}_i = \mathbf{a}_i \otimes \mathbf{h}_i$ acts as identifier for UE i
- **Idea**: Leverage tensor $\mathbf{s}_i$ to inform UE i

HashBeam: Feedback via Downlink Beamforming



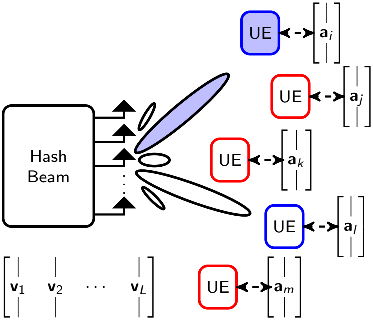
LMMSE beamforming

- Exploit uplink-downlink duality
- $\mathbf{W}_{\mathrm{lmmse}}^{\mathrm{H}} = \left(\alpha^2\sigma^2\mathbf{I} + \mathbf{S}^{\mathrm{H}}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathrm{H}} \in \mathbb{C}^{K\times LM}$
- Base station transmits $\mathbf{v} = \mathbf{W}_{\mathrm{lmmse}}\mathbf{1} \in \mathbb{C}^{LM}$

HashBeam: Feedback via Downlink Beamforming

Decoding feedback at UE

- UE i receives $\mathbf{r}_i = [\langle \mathbf{h}_i, \mathbf{v}_1 \rangle + \mathbf{z}_{i,1}, \ldots, \langle \mathbf{h}_i, \mathbf{v}_L \rangle + \mathbf{z}_{i,L}] \in \mathbb{C}^L$
- UE i decision statistic: $\theta_i = \langle \mathbf{a}_i, \mathbf{r}_i \rangle$
- Khatri-Rao $\mathbf{S} = \mathbf{A} * \mathbf{H}$ and $\mathbf{v} = \mathbf{S} \left(\alpha^2 \sigma^2 \mathbf{I} + \mathbf{S}^{\mathrm{H}} \mathbf{S} \right)^{-1} \mathbf{1}$

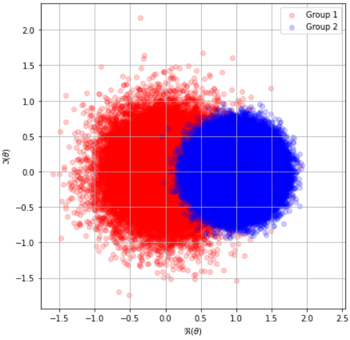


Decision statistic

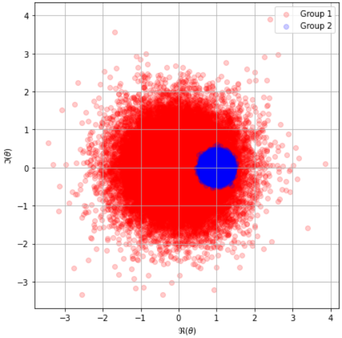
$$\begin{aligned}
\theta_i &= \langle \mathbf{a}_i, \mathbf{r}_i \rangle \\
&= \textstyle\sum_j \left(\mathbf{a}_{i,j}^* \langle \mathbf{h}_i, \mathbf{v}_j \rangle + \mathbf{a}_{i,j}^* \mathbf{z}_{i,j} \right) \\
&= \textstyle\sum_j \left(\langle \mathbf{a}_{i,j} \mathbf{h}_i, \mathbf{v}_j \rangle + \mathbf{a}_{i,j}^* \mathbf{z}_{i,j} \right) \\
&= \langle \mathbf{s}_i, \mathbf{v} \rangle + \langle \mathbf{a}_i, \mathbf{z}_i \rangle
\end{aligned}$$

Compare θ_i to quadratic decision boundary to detect ACK

Structure of Decision Statistic θ



$K = 10,\ M = 10,\ L = 7,$
$\mathrm{SNR} = 5$ dB

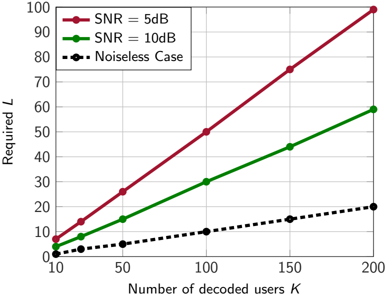


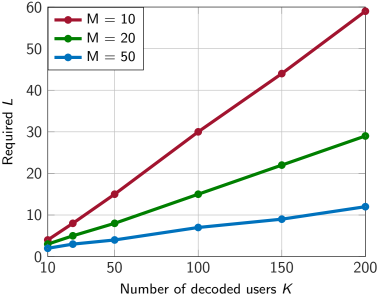
$K = 10,\ M = 5,\ L = 5,$
$\mathrm{SNR} = 15$ dB

Neyman-Pearson approach to quadratic decision region

- Fix $P_{\mathrm{MD}} = 0.05$ and minimize P_{FA}

HashBeam: Feedback via Downlink Beamforming





Performance: required channel uses

- Number of channel uses L scales as $O(K)$
- Adjust L to adapt to any number of antennas M or SNR
- Feedback is individualized - no common feedback signal

Unsourced Random Access – Future Research Avenues

Additional discussion points

- Antennas from one to massive MIMO: In-between?
- Unrolling URA iterative algorithms and better channel models
- Heterogeneous classes of URA devices
- Incremental redundancy for URA: Quest for universality
- Over-the-air federated learning
- Cell-free URA and distributed iterative schemes
- Connection between URA and sketching in TCS

Questions?

A. P. Sabulal and S. Bhashyam. *Joint sparse recovery using deep unfolding With application to massive random access*. ICASSP, 2020.

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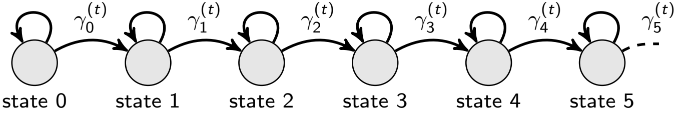
Universal Schemes and Incremental Redundancy

Challenge

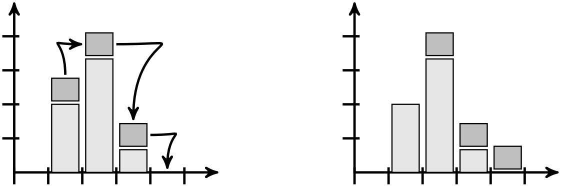
- If device operates in isolation, it does not know total number of active devices K nor slot count for current round
- Packet count and sparse graph should have proper distribution at end of round
- One way to fulfill requirement is for rolling message count to possess proper sparse distribution $\lambda_s(\cdot)$ at every time s

Can this be achieved?

Hybrid ARQ for URA



Hybrid ARQ – Shifting from One Distribution to Another

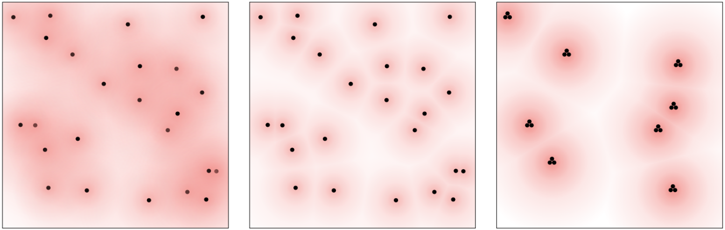


1. Condition 1: Need enough probability mass to push over to neighbor
2. Condition 2: Can't push probability mass past immediate neighbor
3. Conditions can be expressed mathematically in terms of first-order stochastic dominance

$$X \preceq Y \text{ whenever } \Pr(X > m) \leq \Pr(Y > m) \quad \forall m$$

or, equivalently, cumulative distribution function (CDF) of X dominates CDF of Y

Cell-Free Unsourced Random Access

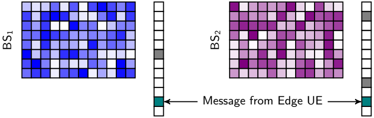


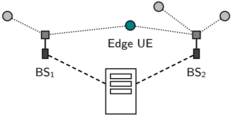
Cell-free MIMO system

Small cell system

Cellular MIMO system

- Expected channel quality as a function of geographical location
- Received power decays, at least, quadratically with distance





Asynchronous UMAC

Asynchronous Neighbor Discovery Using Coupled Compressive Sensing

Vamsi K. Amalladinne, Krishna R. Narayanan, Jean-Francois Chamberland, Dongning Guo

(Submitted on 2 Nov 2018)

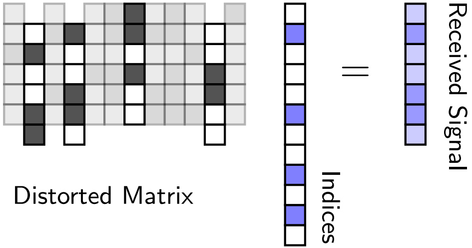
The neighbor discovery paradigm finds wide application in Internet of Things networks, where the number of active devices is orders of magnitude smaller than the total device population. Designing low-complexity schemes for asynchronous neighbor discovery has recently gained significant attention from the research community. Concurrently, a divide-and-conquer framework, referred to as coupled compressive sensing, has been introduced for the synchronous massive random access channel. This work adapts this novel algorithm to the problem of asynchronous neighbor discovery with unknown transmission delays. Simulation results suggest that the proposed scheme requires much fewer transmissions to achieve a performance level akin to that of state-of-the-art techniques.

Subjects: **Signal Processing (eess.SP)**; Information Theory (cs.IT)
Cite as: **arXiv:1811.00687 [eess.SP]**
(or **arXiv:1811.00687v1 [eess.SP]** for this version)

Building Robust Sensing Matrices

- Extending CCS framework with low sample complexity
- Addressing issues pertaining to asynchrony
- Context of neighbor discovery

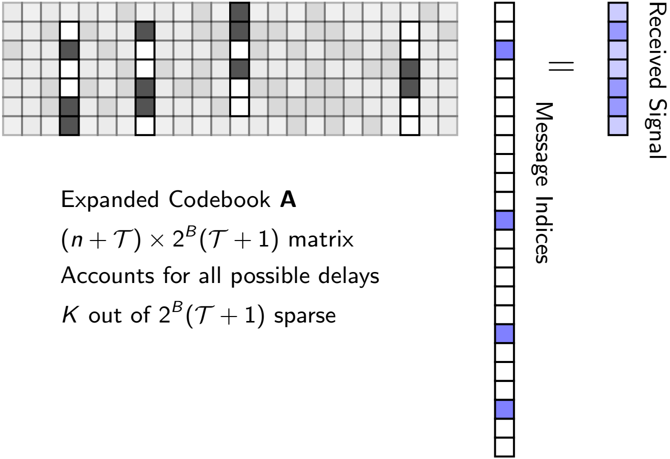
Dealing with Jitter and Asynchrony



Asynchronous Signals

- $\mathbf{y} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{z}$ with $\|\mathbf{x}\|_0 = K$
- $\mathbf{A} \in \mathbb{C}^{(n+\mathcal{T})\times 2^B}$ unknown due to unknown random delays
- Max delay $\mathcal{T}$ known to the decoder

Expanded Codebook through Sensing Matrix



Expanded Codebook $\mathbf{A}$

$(n + \mathcal{T}) \times 2^B(\mathcal{T} + 1)$ matrix

Accounts for all possible delays

K out of $2^B(\mathcal{T} + 1)$ sparse

- Computational complexity of CS solvers: $\mathcal{O}(\mathrm{poly}(2^B(\mathcal{T} + 1)))$

Sparsifying Collision & Data Fragmentation

CHIRRUP: a practical algorithm for unsourced multiple access

Robert Calderbank, Andrew Thompson

(Submitted on 2 Nov 2018)

Unsourced multiple access abstracts grantless simultaneous communication of a large number of devices (messages) each of which transmits (is transmitted) infrequently. It provides a model for machine-to-machine communication in the Internet of Things (IoT), including the special case of radio-frequency identification (RFID), as well as neighbor discovery in ad hoc wireless networks. This paper presents a fast algorithm for unsourced multiple access that scales to 2^{100} devices (arbitrary 100 bit messages). The primary building block is multiuser detection of binary chirps which are simply codewords in the second order Reed Muller code. The chirp detection algorithm originally presented by Howard et al. is enhanced and integrated into a peeling decoder designed for a patching and slotting framework. In terms of both energy per bit and number of transmitted messages, the proposed algorithm is within a factor of 2 of state of the art approaches. A significant advantage of our algorithm is its computational efficiency. We prove that the worst-case complexity of the basic chirp reconstruction algorithm is $\mathcal{O}[nK(\log_2 n + K)]$, where n is the codeword length and K is the number of active users, and we report computing times for our algorithm. Our performance and computing time results represent a benchmark against which other practical algorithms can be measured.

Subjects: **Signal Processing (eess.SP)**
Cite as: **arXiv:1811.00879 [eess.SP]**
(or **arXiv:1811.00879v1 [eess.SP]** for this version)

Submission history
From: Andrew Thompson [view email]
[v1] Fri, 2 Nov 2018 14:25:46 UTC (470 KB)

Which authors of this paper are endorsers? | *Disable MathJax* (*What is MathJax?*)

- Hadamard matrix based compressing scheme with CSS
- Ultra-low complexity decoding algorithm

S. D. Howard, A. R. Calderbank, S. J. Searle. *A fast reconstruction algorithm for deterministic compressive sensing using second order Reed-Muller codes*. CISS 2008

Example: CHIRRUP

- Sensing matrix based on 2nd-order Reed-Muller functions,

$$\phi_{R,b}(a) = \frac{(-1)^{\mathrm{wt}(b)}}{\sqrt{2^m}} i^{(2b+Ra)^T a}$$

  R is binary symmetric matrix with zeros on diagonal, wt represent weight, and $i = \sqrt{-1}$
- Every column of form

$$\begin{matrix} | \\ \mathbf{x}_{R,b} \\ | \end{matrix} = \begin{bmatrix} \phi_{R,b}([0]_2) \\ \phi_{R,b}([1]_2) \\ \vdots \\ \phi_{R,b}([2^m - 1]_2) \end{bmatrix}$$

  $[\cdot]_2$ is integer expressed in radix of 2
- Information encoded into R and b
- **Fast recovery:** Inner-products, Hardmard project onto Walsh basis, get R row column at a time, dechirp, Hadamard project to b

Thank You!

More availabel information at: `https://engprojects.github.io/mMTC/`

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